THE APRIL MEETING IN CHICAGO

The three hundred forty-ninth meeting of the Society was held at the University of Chicago on Friday and Saturday, April 8–9, 1938. The attendance included the following one hundred thirty-six members of the Society:

E. S. Akeley, Leon Alaoglu, A. A. Albert, G. E. Albert, C. B. Allendoerfer, W. L. Ayres, R. W. Babcock, R. H. Bardell, I. A. Barnett, R. C. F. Bartels, Walter Bartky, S. F. Bibb, G. A. Bliss, Henry Blumberg, C. P. Brady, H. R. Brahana, J. L. Brenner, L. E. Bush, W. H. Bussey, H. H. Campaigne, R. D. Carmichael, C. R. Cassity, R. V. Churchill, W. W. S. Claytor, L. W. Cohen, Max Coral, J. J. Corliss, A. P. Cowgill, H. B. Curtis, D. R. Curtiss, W. M. Davis, L. E. Dickson, J. M. Dobbie, J. L. Doob, Ben Dushnik, J. M. Earl, Arnold Emch, H. P. Evans, H S. Everett, L. R. Ford, Bernard Friedman, H. L. Garabedian, H. A. Giddings, J. W. Glover, H. H. Goldstine, G. D. Gore, Cornelius Gouwens, L. M. Graves, L. W. Griffiths, M. B. Haberzetle, P. R. Halmos, W. L. Hart, M. R. Hestenes, T. H. Hildebrandt, J. J. L. Hinrichsen, Václav Hlavatý, D. L. Holl, Ralph Hull, D. H. Hyers, M. H. Ingraham, Dunham Jackson, S. B. Jackson, E. D. Jenkins, Fritz John, M. L. Kales, Chosaburo Kato, I. F. Keeler, R. B. Kershner, J. M. Kinney, W. C. Krathwohl, A. C. Ladner, R. E. Langer, C. G. Latimer, Sam Legvold, R. A. Leibler, M. I. Logsdon, A. C. Lunn, Saunders MacLane, P. T. Maker, Dorothy Manning, W. A. Manning, Ralph Mansfield, H. W. March, Morris Marden, J. R. Mayor, C. W. Mendel, A. N. Milgram, W. L. Mitchell, M. G. Moore, C. W. Moran, D. C. Morrow, E. J. Moulton, I. K. Murray, A. L. Nelson, I. M. Niven, E. A. Nordhaus, E. N. Oberg, G. A. Parkinson, Sam Perlis, H. P. Pettit, G. B. Price, E. D. Rainville, W. C. Randels, W. T. Reid, P. G. Robinson, W. E. Roth, E. H. Rothe, R. G. Sanger, A. C. Schaeffer, M. E. Shanks, C. G. Shover, D. R. Shreve, H. A. Simmons, W. G. Simon, Burke Smith, E. R. Smith, F. C. Smith, S. S. Smith, E. S. Sokolnikoff, I. S. Sokolnikoff, Otto Szász, H. P. Thielman, R. M. Thrall, Leonard Tornheim, W. J. Trjitzinsky, J. I. Vass, H. E. Vaughan, R. W. Wagner, F. P. Welch, R. L. Wilder, F. B. Wiley, K. P. Williams, H. E. Wilson, L. A. Wolf, M. C. Wolf, F. E. Wood.

The Friday morning session was in three sections, the first devoted to Geometry, the second to Applied Mathematics and Algebra, and the third to Analysis. On Friday afternoon Professor C. G. Latimer, of the University of Kentucky, delivered the Symposium Lecture on *The arithmetics of generalized quaternions*. A general session was held on Saturday morning.

The dinner was held at Judson Court on Friday evening and was attended by ninety-five persons. Professor E. J. Moulton, of Northwestern University, acted as toastmaster. Professor G. A. Bliss, of the University of Chicago, spoke of the coming International Congress of Mathematicians and called on Professor Václav Hlavatý, of the Karl University of Prague, for a few remarks. Professor Dunham Jackson, of the University of Minnesota, told of the arrangements for

the Semicentennial of the Society. Professor M. H. Ingraham, of the University of Wisconsin, as retiring secretary of the meeting, expressed the appreciation of the Society for the hospitality of the University of Chicago and the excellent local arrangements made by Dr. R. G. Sanger during the past ten years.

On Friday morning Professor W. H. Bussey presided in the section for Geometry, Professor M. H. Ingraham in the section for Applied Mathematics and Algebra, and Professor R. E. Langer in the section for Analysis. Professor A. A. Albert presided at the Symposium Lecture. At the general session on Saturday morning Professors G. A. Bliss and T. H. Hildebrandt presided.

Titles and cross references to abstracts of papers presented at this meeting are given below. Papers numbered 1 to 9 were read before the section for Geometry, those numbered 10 to 21 before the section for Applied Mathematics and Algebra, those numbered 22 to 32 before the section for Analysis, and those numbered 33 to 43 before the general session. Papers numbered 44 to 51, whose abstract numbers are followed by the letter *t*, were read by title. Dr. J. W. T. Suckau was introduced by Professor Tibor Radó, Dr. Olaf Helmer by Professor A. A. Albert, and Dr. R. J. Duffin by Dr. A. C. Schaeffer. Paper number 2 was read by Professor Ford, 11 by Professor Sokolnikoff, 32 by Dr. Schaeffer, 38 by Dr. Claytor, and 41 by Dr. Randels.

1. *The maps determined by the principal curves associated with five and six points in the plane*, by C. R. Cassity. (Abstract 44-3-137.)

2. *The isotenic circle*, by J. W. Hahn and L. R. Ford. (Abstract 44-5-204.)

3. *New properties of cubic surfaces*, by Arnold Emch. (Abstract 44-3-141.)

4. *An indicatrix for Riemann spaces*, by C. B. Allendoerfer. (Abstract 44-5-169.)

5. *The four-vertex theorem on the sphere*, by S. B. Jackson. (Abstract 44-5-219.)

6. *Theory of the topological order in some linear topological spaces* (preliminary report), by E. H. Rothe. (Abstract 44-5-257.)

7. *Concerning continuous linear orders*, by Ben Dushnik. (Abstract 44-5-193.)

8. *Note on ergodic curves*, by R. B. Kershner. (Abstract 44-5-229.)

9. *A lemma on squares*, by J. W. T. Suckau. (Abstract 44-5-267.)

10. *Saint-Venant's flexure problem for a regular polygon* (preliminary report), by R. C. F. Bartels. (Abstract 44-5-175.)

11. *Thermal stresses in elastic plates*, by I. S. Sokolnikoff and Elizabeth S. Sokolnikoff. (Abstract 44-5-266.)

12. *Generating functions associated with stochastic processes*, by E. S. Akeley. (Abstract 44-5-281.)

13. *Integral functions with coefficients in a given field* (preliminary report), by Olaf Helmer. (Abstract 44-5-212.)

14. *Apolarity of trilinear forms and pencils of bilinear forms*, by R. M. Thrall. (Abstract 44-3-159.)

15. *On the continuity of certain matrix functions* (preliminary report), by R. W. Wagner. (Abstract 44-5-275.)

16. *Integral sets of quaternion algebras over a function field*, by Leonard Tornheim. (Abstract 44-5-270.)

17. *On the representation of periodic residue systems modulo m*, by E. D. Rainville. (Abstract 44-5-252.)

18. *The properties of finite hypergroups*, by H. H. Campaigne. (Abstract 44-5-183.)

19. *Structure elements of quasi-groups*, by R. J. Duffin. (Abstract 44-5-192.)

20. *Two new universal Waring theorems*, by Mary B. Haberzetle. (Abstract 44-5-203.)

21. *A Waring problem*, by I. M. Niven. (Abstract 44-5-246.)

22. *A generalized Lambert series*, by J. M. Dobbie. (Abstract 44-3-139.)

23. *The convergence of polynomials on an infinite interval*, by J. M. Earl. (Abstract 44-5-194.)

24. *A Tauberian theorem related to Leroy summability*, by M. L. Kales. (Abstract 44-5-227.)

25. *Weak convergence of linear functionals*, by Leon Alaoglu. (Abstract 44-3-135.)

26. *A generalization of the problem of Bolza in the calculus of variations*, by H. H. Goldstine. (Abstract 44-5-200.)

27. *The inequalities of Morse when the maximum type is at most three*, by R. H. Bardell. (Abstract 44-5-174.)

28. *A note on convex functions*, by Bernard Friedman. (Abstract 44-5-198.)

29. *A mean value theorem for second order linear differential equations with constant coefficients*, by Fritz John. (Abstract 44-5-224.)

30. *Summability of expansions in solutions of differential equations*, by M. G. Moore. (Abstract 44-5-240.)

31. *Non-linear singular differential equations containing a parameter*, by F. P. Welch. (Abstract 44-3-163.)

32. *On some inequalities of S. Bernstein and W. Markoff on the derivatives of polynomials*, by A. C. Schaeffer and R. J. Duffin. (Abstract 44-5-258.)

33. *On the units of indefinite quaternion algebras*, by Ralph Hull. (Abstract 44-5-217.)

34. *On cyclic algebras*, by A. A. Albert. (Abstract 44-3-136.)

35. *On the matrix equation* $TA = BT$, by M. H. Ingraham. (Abstract 44-5-218.)

36. *Transcendence degrees and p-bases in terms of non-modular lattices*, by Saunders MacLane. (Abstract 44-3-149.)

37. *The boundary problem of linear differential systems in the complex domain*, by R. E. Langer. (Abstract 44-5-285.)

38. *Homogeneous locally connected continua.* I, by W. S. Claytor and R. L. Wilder. (Abstract 44-5-186.)

39. *A theory of critical points of functions in the calculus of variations*, by M. R. Hestenes. (Abstract 44-5-213.)

40. *On Fourier series with restricted coefficients*, by Otto Szász. (Abstract 44-5-269.)

41. *On summability by Riesz means*, by H. L. Garabedian and W. C. Randels. (Abstract 44-5-199.)

42. *Maximal orders in rational cyclic algebras of composite degree*, by Sam Perlis. (Abstract 44-5-286.)

43. *The group of automorphisms of the abelian group of order* p^{nr} *and type* $(r, r, \cdots, r)$, by J. L. Brenner. (Abstract 44-5-181.)

44. *On volumes bounded by cylindrical surfaces*, by J. D. Mancill. (Abstract 44-3-151-*t*.)

45. *A note on the asymptotic Waring problem for homogeneous polynomial summands*, by Harold Chatland. (Abstract 44-3-138-*t*.)

46. *Concerning R. L. Moore's Axiom* 5, by F. B. Jones. (Abstract 44-3-146-*t*.)

47. *Sums of nth powers in fields of prime characteristic*, by Leonard Tornheim. (Abstract 44-5-271-*t*.)

48. *A note on normal division algebras of prime degree*, by A. A. Albert. (Abstract 44-5-168-*t*.)

49. *Axioms for lattices connected with dependence relations*, by Saunders MacLane. (Abstract 44-5-234-*t*.)

50. *Derived periodic residue systems modulo p*, by E. D. Rainville. (Abstract 44-5-251-*t*.)

51. *A note on cubic graphs*, by E. W. Miller. (Abstract 44-5-238-*t*.)

W. L. AYRES,
Associate Secretary

THE APRIL MEETING IN BERKELEY

The three hundred fiftieth meeting of the American Mathematical Society was held at the University of California on Saturday, April 9, 1938. The morning and afternoon sessions were presided over at various times by Professors W. M. Whyburn, H. F. Blichfeldt, Frederick Wood, and G. C. Evans. During the morning session an address entitled *General differential geometries and related topics* was delivered by Professor A. D. Michal, of the California Institute of Technology. Luncheon for members and their guests was served at the Faculty Club. The attendance was about eighty, including the following fifty-three members of the Society:

H. A. Arnold, H. M. Bacon, G. A. Baker, E. T. Bell, B. A. Bernstein, H. F. Blichfeldt, C. R. Bubb, F. A. Butter, P. H. Daus, A. H. Diamond, R. P. Dilworth, C. H. Dix, John Dyer-Bennet, G. C. Evans, A. L. Foster, H. E. Goheen, E. C. Goldsworthy, J. W. Green, D. W. Hall, N. A. Hall, O. G. Harrold, R. M. Hermes, P. G. Hoel, J. W. Hurst, Frank Irwin, R. D. James, S. H. Levy, Hans Lewy, A. T. Lonseth, C. F. Luther, A. D. Michal, F. R. Morris, W. H. Myers, C. A. Noble, T. M. Putnam, R. M. Robinson, E. B. Roessler, E. L. Ross, Henry Scheffé, P. M. Singer, Pauline Sperry, L. H. Swinford, H. L. Turrittin, S. E. Urner, J. V. Uspensky, R. K. Wakerling, Morgan Ward, D. E. Whelan, W. M. Whyburn, A. R. Williams, B. C. Wong, Frederick Wood, Max Zorn.

Titles of papers read at the meeting follow. Those whose abstract numbers are followed by the letter *t* were read by title.

1. *Correlation surfaces of two or more indices when the components of the indices are normally distributed*, by G. A. Baker. (Abstract 44-5-173.)

2. *The solution of the trinomial equation in infinite series by the method of iteration*, by N. A. Hall. (Abstract 44-5-207.)

3. *Remarks on Laplace's solution of the problem of the attraction of ellipsoids*, by J. V. Uspensky. (Abstract 44-5-274.)

4. *The distribution of integers represented by quadratic forms*, by R. D. James. (Abstract 44-5-223.)

5. *On a decomposition of true cyclic elements*, by D. W. Hall. (Abstract 44-5-205.)

6. *A new representation of real numbers*, by A. L. Foster. (Abstract 44-5-196.)

7. *Residuated lattices*, by Morgan Ward and R. P. Dilworth. (Abstract 44-5-276.)

8. *Approximate distribution of the zeros of certain exponential sums*, by H. L. Turrittin. (Abstract 44-5-273.)

9. *The χ^2 distribution for small samples*, by P. G. Hoel. (Abstract 44-5-215.)

10. *Matrix expressions of Hooke's law*, by C. H. Dix. (Abstract 44-5-189.)

11. *Non-commutative residuation* (preliminary report), by R. P. Dilworth. (Abstract 44-5-188.)

12. *Correlations in terms of central collineations and central correlations*, by P. H. Daus. (Abstract 44-5-187.)

13. *Fixed-point theorems in semi-ordered spaces*, by H. A. Arnold. (Abstract 44-5-170.)

14. *Sets of postulates for Boolean groups*, by B. A. Bernstein. (Abstract 44-5-177.)

15. *On a lemma about matrices*, by Max Zorn. (Abstract 44-5-280.)

16. *A differential equation of R. Liouville*, by L. H. Swinford. (Abstract 44-5-268.)

17. *Harmonic functions in domains with multiple boundary points*, by J. W. Green. (Abstract 44-5-202.)

18. *On ternary forms with non-vanishing Hessian and on spherical harmonics*, by Hans Lewy. (Abstract 44-5-231.)

19. *An increase in the lower bound of possible solutions of the Fermat equation*, by Glenn James. (Abstract 44-5-222-*t*.)

20. *Generalized Stirling transforms of sequences*, by E. T. Bell. (Abstract 44-5-176-*t*.)

21. *The differential in abstract algebra*, by A. D. Michal. (Abstract 44-5-237-*t*.)

22. *Differentials in linear topological spaces*, by A. D. Michal. (Abstract 44-5-235-*t*.)

23. *Differential geometries with linear topological coordinates*, by A. D. Michal. (Abstract 44-5-236-*t*.)

24. *Symmetrical form for the general strain matrix*, by C. H. Dix. (Abstract 44-5-190-*t*.)

25. *The theory of integration in linear L-spaces*, by H. A. Arnold. (Abstract 44-5-171-*t*.)

T. M. PUTNAM,
Associate Secretary

THE APRIL MEETING IN CHARLOTTESVILLE

The three hundred fifty-first meeting of the American Mathematical Society was held at the University of Virginia on Friday and Saturday, April 15–16, 1938. The attendance included the following one hundred seventy-two members of the Society:

Beatrice Aitchison, H. F. Archibald, W. L. Ayres, Reinhold Baer, M. F. Becker, G. D. Birkhoff, Garrett Birkhoff, Archie Blake, R. P. Boas, H. F. Bohnenblust, Richard Brauer, B. L. Brown, E. T. Browne, R. S. Burington, Herbert Busemann, J. H. Bushey, J. Hughes Bushey, J. W. Calkin, Leonard Carlitz, A. F. Carpenter, H. C. Carter, C. E. Clark, J. A. Clarkson, A. H. Clifford, Nancy Cole, T. F. Cope, M. J. Cox, H. S. M. Coxeter, H. B. Curry, J. H. Curtiss, D. B. DeLury, Douglas Derry, T. L. Downs, Arnold Dresden, F. G. Dressel, D. M. Dribin, E. J. Finan, I. G. Foster, C. W. Franklin, Philip Franklin, A. H. Frink, Orrin Frink, T. C. Fry, A. J. Garland, T. S. George, J. J. Gergen, J. W. Givens, B. D. Grant, Margaret Gurney, B. L. Hagen, Marshall Hall, P. R. Halmos, Harris Hancock, Philip Hartman, Erich Hecke, E. H. C. Hildebrandt, Einar Hille, M. P. Hollcroft, T. R. Hollcroft, Witold Hurewicz, M. H. Ingraham, Nathan Jacobson, Walter Jennings, Fritz John, F. E. Johnston, B. W. Jones, H. A. Jordan, E. R. van Kampen, B. C. Keeler, J. L. Kelley, J. R. F. Kent, J. R. Kline, M. S. Knebelman, H. L. Krall, O. E. Lancaster, E. P. Lane, G. A. Larew, J. W. Lasley, Solomon Lefschetz, A. M. M. Lehr, B. Z. Linfield, Marie Litzinger, E. R. Lorch, W. V. Lovitt, J. J. Luck, R. S. McKee, E. J. McShane, E. L. Mackie, H. M. MacNeille, P. T. Maker, M. H. Martin, Walther Mayer, F. M. Mears, Karl Menger, A. K. Mitchell, W. C. Mitchell, Virginia Modesitt, C. N. Moore, E. M. Morenus, C. B. Morrey, Marston Morse, D. C. Murdoch, F. D. Murnaghan, Tadasi Nakayama, C. J. Nesbitt, D. L. Netzorg, John von Neumann, E. J. Oglesby, Oystein Ore, J. C. Oxtoby, F. W. Owens, H. B. Owens, B. J. Pettis, C. S. Pettis, R. S. Phillips, W. T. Puckett, H. A. Rademacher, W. C. Randels, S. E. Rasor, F. W. Reed, M. S. Rees, C. N. Reynolds, Moses Richardson, R. G. D. Richardson, E. K. Ritter, J. H. Roberts, G. de B. Robinson, O. F. G. Schilling, E. R. Schneckenburger, I. J. Schoenberg, G. E. Schweigert, Wladimir Seidel, R. G. Simond, Abraham Sinkov, Frank Smithies, Virgil Snyder, C. M. Sparrow, J. M. Stetson, R. W. Stokes, M. H. Stone, E. C. Strayhorn, J. L. Synge, J. D. Tamarkin, J. H. Taylor, M. E. Taylor, J. M. Thomas, Arthur Tilley, E. W. Titt, C. B. Tompkins, M. M. Torrey, W. J. Trjitzinsky, A. W. Tucker, J. W.Tukey, G. B. Van Schaack, Oswald Veblen, R. J. Walker, A. D. Wallace, Henry Wallman, S. E. Warschawski, C. W. Watkeys, Albert Wertheimer, Hermann Weyl, A. H. Wheeler, A. P. Wheeler, C. H. Wheeler, G. T. Whyburn, Norbert Wiener, C. W. Williams, John Williamson, Audrey Wishard, Oscar Zariski, H. S. Zuckerman.

The meeting opened Friday morning in two sections, Topology, Professors J. H. Roberts and M. S. Knebelman presiding; and Analysis, Professor E. T. Browne presiding.

Friday afternoon was devoted to a symposium on *Lattice theory* at which the following addresses were given: *The known applications of lattices*, by Mr. Garrett Birkhoff; *Structures and algebraic applications*,

by Professor Oystein Ore; *Representations of Boolean algebras*, by Professor M. H. Stone. The discussion was led by Professor Reinhold Baer, Dr. H. M. MacNeille, Professor Karl Menger, and Professor John von Neumann. Professor Marston Morse presided at this meeting.

On Saturday morning, there were again two sections, Analysis and Geometry, and Algebra. The presiding officers were Professors Philip Franklin and Oystein Ore.

All sessions were held in Monroe Hall, University of Virginia.

On Friday afternoon, a tea for visiting ladies was given at the home of Dean and Mrs. J. S. Wilson. The buffet dinner Friday evening at the Farmington Country Club was attended by two hundred seven members and guests. Many small parties visited Monticello and Ash Lawn at various times.

The Society adopted a resolution, presented by Professor Philip Franklin, expressing its appreciation and thanks to the President of the University of Virginia and the members of the Department of Mathematics for their competent arrangements and cordial hospitality.

Meetings of various committees of the International Congress of Mathematicians were held, and important decisions concerning the program of the Congress were made.

The Council met on Saturday, April 16, at 12:30 P.M. in the Club Room of the Monticello Hotel. The report of this meeting follows.

The Secretary announced the election of the following seventy-one persons to membership in the Society:

Mr. Frank Albert Alfieri, Metropolitan Life Insurance Company, New York, N. Y.;
Mr. DeWitt Dukes Barlow, Atlantic, Gulf and Pacific Company, New York, N. Y.;
Mr. Stanley Urner Benscoter, U. S. Engineers Office, Kansas City, Mo.;
Mr. Robert Martin Besancon, Chicago, Ill.;
Mr. Peter B. Boyer, Oregon Institute of Technology;
Mr. Arthur Albert Francis Brown, Princeton University;
Dr. Frederick Winfield Brown, University of Kansas City;
Miss Helen Elizabeth Clarkson, Jacksonville College, Jacksonville, Tex.;
Mr. William Christopher Coombs, Colonial Radio Corporation, Buffalo, N. Y.;
Professor William Alonzo Cordrey, Lambuth College, Jackson, Tenn.;
Mr. Lester L. Cronvich, Tulane University;
Mr. Maurice Rolland Demers, Brown University;
Professor Paul Arnold De Vore, Central Missouri State Teachers College;
Mr. Philip DiSalvatore, Princeton University;
Mr. James Francis Downey, Charles E. Bedaux Company of New York, New York, N. Y.;
Mr. Clayburn Combes Elder, Metropolitan Water District of Southern California, Los Angeles, Calif.;

Mr. Dean Burritt Ellis, Arkansas State College;
Miss Mary Louise Elveback, University of Minnesota;
Dr. Charles Stever Fazel, The Solvay Process Company, Hopewell, Va.;
Mr. Frederick Arthur Ficken, Lafayette College;
Mr. George Fisanick, Metropolitan Life Insurance Company, New York, N. Y.;
Professor Harry T. Fleddermann, Loyola University, New Orleans, La.;
Miss Helen Gilchrist Fudge, Holmes Junior High School, Philadelphia, Pa.;
Dean Godofredo García, University of San Marcos, Lima, Peru;
Professor Frank C. Gentry, University of Oklahoma;
Mr. Robert Ellsworth Glover, U. S. Bureau of Reclamation, Denver, Colo.;
Miss Lilah Genevieve Godfrey, Walla Walla College;
Miss Bernice Duncan Grant, Alderson Broaddus College, Philippi, W. Va.;
Professor Edward L. Hill, University of Minnesota;
Mr. Louis Ricketts Hodell, Carter Oil Company, Tulsa, Okla.;
Dr. Stanley Bartlett Jackson, University of Wisconsin;
Sister James Stanislaus, St. Teresa Junior College, Kansas City, Mo.;
Lieutenant Maximiano S. Janairo, U. S. Army, Philippine Department, Manila, P. I ;
Mr. Walter Jennings, Ohio State University;
Professor Howard Kenneth Justice, University of Cincinnati;
Professor Benjamin C. Keeler, Webb Institute, New York, N. Y.;
Professor Robert Hilton Knox, Virginia Military Institute;
Dr. Joseph F. Kubis, Fordham University;
Dr. Ruth Onetta Lane, University High School, Iowa City, Iowa;
Mr. Coe E. Leach, Green Camp, Ohio;
Mr. Talmage Hoyle Lee, University of Wisconsin;
Brother Leo (Daniel H. Quinn), St. Francis College, Brooklyn, N. Y.;
Mr. William John Martyn, Otago Boys' High School, Dunedin, New Zealand;
Sister M. St. Alphonsus, Immaculata College, Immaculata, Pa.;
Sister Mary Amator, University of Pennsylvania;
Professor John Neville Meighan, Storer College, Harper's Ferry, W. Va.;
Mr. Julius Sumner Miller, Dillard University, New Orleans, La.;
Professor Carey Gardner Mumford, North Carolina State College;
Professor Carl Wallace Munshower, Colgate University;
Miss Irene Kathryn Murray, Chicago, Ill.;
Mr. Howard M. Nahikian, North Carolina State College;
Professor John Lester Nickerson, Mount Allison University, Sackville, N. B.;
Mr. John M. H. Olmstead, Princeton University;
Mr. Edson Ruther Peck, Evanston, Ill.;
Dr. Irwin Earl Perlin, Armour Institute of Technology;
Mr. Harry Polachek, Haaren High School, New York, N. Y.;
Reverend Joseph G. Précourt, Providence College;
Miss Adrienne Sophie Rayl, E. D. White High School, New Orleans, La.;
Mr. John Karlem Riess, Alcee Fortier High School, New Orleans, La.;
Professor David Anton Frederic Robinson, University of Toronto;
Professor Emily Lee Ross, University of Nevada;
Professor John Edward Sandt, Marietta College;
Professor Edward S. Smith, University of Cincinnati;
Professor Claude Newton Stokes, Temple University;
Mr. Paul Chandler Sweetland, Garden City Junior College, Garden City, Kan.;
Mr. Emil O. Tolonen, Highland Park, Mich.;

Dr. Robert McDowell Thrall, University of Michigan;
Mr. Ralph Edgar Traber, Princeton University;
Mr. John Byron Tracy, Taunton, Mass.;
Professor Charles Walmsley, Dalhousie University, Halifax, N. S.;
Miss Audrey Wishard, Radcliffe College.

It was reported that the following had been elected as nominees on the Institutional Memberships of the institutions and departments indicated:

Bryn Mawr College: Miss Elizabeth Monroe.
Division of Mathematics, Harvard University: Dean Harald Malcolm Westergaard.
Indiana University: Dr. Frank Kelley Edmondson.
The Johns Hopkins University: Miss Almeda Jane Garland and Dr. David Leon Netzorg.
Lehigh University: Mr. Henry Louis Langhaar.
Mathematical Association of America: Professor Juna Lutz Beal.
University of Michigan: Messrs. Herman Edmund Arnold, John Dennis Fitzpatrick, Robert Eugene Gaskell, Carl Frederick Kossack, and Elbridge Putnam Vance.
University of Notre Dame: Reverend Frederick Michael Gassensmith.

The following appointments were announced: as representative of the Society at a conference on problems in publication of research held under the auspices of the American Philosophical Society in Philadelphia, February 18–19, 1938, Professor J. D. Tamarkin; as representative at a group of conferences on higher education and the dedication of new buildings at Louisiana State University on April 6–8, 1938, Professor S. T. Sanders; as representative at the celebration of the one hundredth anniversary of the chartering of Greensboro College on May 20–21, 1938, Professor Ruth W. Stokes; and as representative at the dedication of The Franklin Institute of the State of Pennsylvania on May 19–21, 1938, Professor J. R. Kline.

Two changes in the personnel of the Committee on the Award of the Bôcher Prize in Analysis were reported: Professor T. H. Hildebrandt (chairman) and Dean R. D. Carmichael were appointed in the places of Dean G. D. Birkhoff and Professor G. A. Bliss, respectively, who had resigned.

The following dates and places were chosen for future meetings: New York City, February 25, 1939; Durham, North Carolina, April 7–8, 1939, in connection with the Centennial of Trinity College (now Duke University) and the proposed symposium on mathematics; Columbus, Ohio, Annual Meeting of 1939, in connection with the meetings of the American Association for the Advancement of Science.

The following Committee on Arrangements for the meeting at Duke University, April 7–8, 1939, was appointed: Professors J. J. Gergen (chairman), Leonard Carlitz, T. R. Hollcroft, J. H. Roberts,

G. T. Whyburn. Professors J. J. Gergen and J. D. Tamarkin were appointed a committee on symposium for this meeting.

On the recommendation of the Committee on the Gibbs Lecture, the Council voted to omit this lecture in 1938.

The Colloquium Committee reported that the book *Orthogonal Polynomials*, by Professor Gabor Szegö, has been accepted for publication in the Colloquium Series.

Titles and cross references to the abstracts of the papers read at this meeting follow below. The papers were read in the various sections as follows: papers numbered 1 to 8, Topology; papers 9 to 16, Analysis; papers 17 to 28, Analysis and Geometry; papers 29 to 40, Algebra. Papers 41 to 70, whose abstract numbers are followed by the letter *t*, were read by title. Paper 2 was read by Mr. Oxtoby, 4 by Professor Roberts, 10 by Dr. Aline Frink, 12 by Professor Morse, 22 by Professor Hermann Weyl, 25 by Professor Snyder, and 32 by Professor Nakayama.

1. *A metric related to property S*, by J. L. Kelley. (Abstract 44-3-111.)

2. *The existence of metrically transitive transformations* (preliminary report), by J. C. Oxtoby and S. M. Ulam. (Abstract 44-5-248.)

3. *Concerning local connectedness under the inverse of certain continuous transformations*, by W. T. Puckett. (Abstract 44-3-154.)

4. *Monotone transformations on 2-dimensional manifolds*, by J. H. Roberts and N. E. Steenrod. (Abstract 44-5-255.)

5. *On point-wise almost periodic homeomorphisms*, by W. L. Ayres. (Abstract 44-5-172.)

6. *Chain-deformation retracts* (preliminary report), by A. W. Tucker. (Abstract 44-5-272.)

7. *On 0-regular transformations*, by A. D. Wallace. (Abstract 44-3-161.)

8. *Segmental curves and faceted surfaces*, by F. W. Reed. (Abstract 44-5-253.)

9. *On the Stieltjes moment problem* (preliminary report), by R. P. Boas (National Research Fellow). (Abstract 44-5-179.)

10. *Polygonal variations*, by Aline H. Frink and Orrin Frink. (Abstract 44-3-142.)

11. *Maximal real determinants*, by Philip Franklin. (Abstract 44-5-197.)

12. *Symbolic dynamics*, by Marston Morse and G. A. Hedlund. (Abstract 44-5-242.)

13. *An application of Hurwitz's convergence factor theorem to the generalization of Poisson's summation of Fourier series*, by C. N. Moore. (Abstract 44-5-239.)

14. *On the number of abelian groups of a given order*, by D. L. Netzorg. (Abstract 44-5-245.)

15. *On certain differential equations for Tchebycheff polynomials*, by H. L. Krall. (Abstract 44-3-147.)

16. *Some existence theorems in the calculus of variations.* II. *Isoperimetric problems*, by E. J. McShane. (Abstract 44-5-233.)

17. *Invariants of certain stochastic transformations: the mathematical theory of gambling systems*, by P. R. Halmos. (Abstract 44-5-208.)

18. *The differentiability of functions minimizing certain double integrals*, by C. B. Morrey. (Abstract 44-5-241.)

19. *Conditions on $u(x, y)$ and $v(x, y)$ necessary and sufficient for the regularity of $u+iv$*, by P. T. Maker. (Abstract 44-3-150.)

20. *Metric spaces and completely monotone functions*, by I. J. Schoenberg. (Abstract 44-5-260.)

21. *Notes on linear transformations.* II. *Analyticity of semi-groups*, by Einar Hille. (Abstract 44-5-214.)

22. *Meromorphic curves*, by Hermann Weyl and F. J. Weyl. (Abstract 44-5-277.)

23. *A plane projection of the polytope 2_{21} whose 27 vertices represent the lines on the general cubic surface*, by H. S. M. Coxeter. (Abstract 44-3-98.)

24. *Groups of inscribed tetrahedra*, by A. H. Wheeler. (Abstract 44-5-278.)

25. *A Cremona involution in S_3 without a surface of invariant points*, by Virgil Snyder and Evelyn Carroll-Rusk. (Abstract 44-5-265.)

26. *Involutory systems of curves on ruled surfaces*, by A. F. Carpenter. (Abstract 44-5-184.)

27. *Contact transformations*, by M. S. Knebelman. (Abstract 44-5-230.)

28. *Plane curve systems associated with singular surfaces*, by T. R. Hollcroft. (Abstract 44-5-216.)

29. *Prüfer ideals in commutative rings*, by D. M. Dribin (National Research Fellow). (Abstract 44-5-191.)

30. *Normal semi-linear transformations*, by Nathan Jacobson. (Abstract 44-5-220.)

31. *On groups of linear transformations*, by Richard Brauer. (Abstract 44-5-180.)

32. *Note on symmetric algebras*, by Tadasi Nakayama and C. J. Nesbitt. (Abstract 44-5-244.)

33. *Quasi-groups which satisfy certain generalized associative laws*, by D. C. Murdoch. (Abstract 44-5-243.)

34. *Identities analogous to Ramanujan's identities involving the par-*

tition function, by H. S. Zuckerman (National Research Fellow). (Abstract 44-5-250.)

35. *Matrices normal with respect to an hermitian matrix*, by John Williamson. (Abstract 44-3-165.)

36. *Related genera of positive ternary quadratic forms*, by B.W. Jones. (Abstract 44-5-225.)

37. *The generalized Clebsch-Gordan series*, by F. D. Murnaghan. (Abstract 44-3-152.)

38. *Uniformization in valuation rings and resolutions of singularities of an algebraic surface*, by Oscar Zariski. (Abstract 44-5-279.)

39. *Sequences defined by non-linear algebraic difference equations*, by O. E. Lancaster. (Abstract 44-3-148.)

40. *Equidistribution of residues in sequences*, by Marshall Hall. (Abstract 44-5-206.)

41. *On oscillations of real sequences and of their transforms by square matrices*, by R. P. Agnew. (Abstract 44-5-167-*t*.)

42. *New properties of lattices*, by Garrett Birkhoff. (Abstract 44-5-178-*t*.)

43. *On the classification of correlations in space*, by E. T. Browne. (Abstract 44-5-182-*t*.)

44. *An accurate method for obtaining the derivative function from observational data*, by J. W. Cell. (Abstract 44-5-185-*t*.)

45. *A continuous group of contact transformations containing the generalized pedal transformation*, by J. M. Feld. (Abstract 44-5-195-*t*.)

46. *A generalization of multiple sequence transformations*, by H. J. Hamilton. (Abstract 44-5-210-*t*.)

47. *Change of dimension in sequence transformations*, by H. J. Hamilton. (Abstract 44-5-209-*t*.)

48. *Concerning collections of continua in a separable space which do not cross*, by O. H. Hamilton. (Abstract 44-5-211-*t*.)

49. *The half-group of cosets belonging to a group*, by Charles Hopkins. (Abstract 44-3-145-*t*.)

50. *Simple Lie algebras over a field of characteristic zero*, by Nathan Jacobson. (Abstract 44-5-221-*t*.)

51. *Perfect subdivision of surfaces*, by I. N. Kagno. (Abstract 44-5-226-*t*.)

52. *The canonical reduction of Hamiltonian systems*, by E. R. van Kampen and Aurel Wintner. (Abstract 44-5-228-*t*.)

53. *Approximation to algebraic numbers by means of periodic sequences of transformations on quadratic forms*, by R. R. McDaniel. (Abstract 44-5-232-*t*.)

54. *The analysis of the representations of the real n-dimensional or-*

thogonal group induced by irreducible, rational, integral representations of the full linear group, by F. D. Murnaghan. (Abstract 44-3-153-*t*.)

55. *Representation and equivalence of forms*, by Rufus Oldenburger. (Abstract 44-5-247-*t*.)

56. *Concerning the 1-dimensional Betti group under interior transformations*, by W. T. Puckett. (Abstract 44-3-155-*t*.)

57. *A new proof of two of Ramanujan's identities*, by H. A. Rademacher and H. S. Zuckerman (National Research Fellow). (Abstract 44-5-249-*t*.)

58. *Systems of differential equations. I. Theory of ideals*, by J. F. Ritt. (Abstract 44-5-254-*t*.)

59. *Piecemeal univalency of analytic functions*, by M. S. Robertson. (Abstract 44-3-156-*t*.)

60. *On the integration of Wilczynski's equations by quadratures*, by L. B. Robinson. (Abstract 44-5-256-*t*.)

61. *A generalization of local class field theory*, by O. F. G. Schilling. (Abstract 44-5-259-*t*.)

62. *On the Peano curve of Lebesgue*, by I. J. Schoenberg. (Abstract 44-5-261-*t*.)

63. *On a certain class of continued fractions with applications to Bessel functions and Lommel polynomials*, by H. M. Schwartz. (Abstract 44-5-262-*t*.)

64. *An example in conformal mapping*, by Wladimir Seidel. (Abstract 44-5-263-*t*.)

65. *Degree of approximation by polynomials in z and 1/z*, by W. E. Sewell. (Abstract 44-3-157-*t*.)

66. *Note on the relation between Lipschitz conditions and degree of polynomial approximation*, by W. E. Sewell. (Abstract 44-3-158-*t*.)

67. *On the distribution of the zeros of orthogonal polynomials*, by J. A. Shohat. (Abstract 44-5-264-*t*.)

68. *On local homeomorphisms*, by A. D. Wallace. (Abstract 44-3-160-*t*.)

69. *On regular transformations*, by A. D. Wallace. (Abstract 44-3-162-*t*.)

70. *Interior surface transformations*, by G. T. Whyburn. (Abstract 44-3-164-*t*.)

T. R. Hollcroft,
Associate Secretary

ALBERT ON MODERN HIGHER ALGEBRA

Modern Higher Algebra. By A. A. Albert. (The University of Chicago Science Series.) Chicago, The University of Chicago Press, 1937. 14+319 pp.

Since the time of Dedekind it has become increasingly evident that the most satisfactory way of dealing with algebraic questions is to found the whole structure of algebra abstractly on certain fundamental abstract concepts. These are the concepts of group, ring, and field. The book we are considering here is apparently the first one in English to carry out this program. The contents of the book may be briefly summarized as follows: 1. Foundations; theory of groups, rings, fields (Chapter 1, 2). 2. Matrices and matrix algebras (Chapters 3, 4, 5, 10). 3. Galois theory (Chapters 6, 7, 8, 9). 4. Theory of valuation (Chapters 11, 12).

1. The book begins with a discussion of groups and their simpler properties and then goes on to define what is called a *ring*. A ring is an additive group in which multiplication is defined so as to be associative (but not necessarily commutative) and distributive with respect to addition. A fundamental idea here is that of *equivalent* groups and *equivalent* rings, the term "equivalent" being synonymous with the more commonly used "isomorphic." The most important concept is the construction of the polynomial ring in a scalar indeterminate over a given ring; the treatment of this subject on pages 18–19 seems somewhat confusing, although it is fundamentally sound.

Next are treated special kinds of rings, notably *integral domains*, which are commutative rings with an identity element and no zero divisors, and *fields*, which are integral domains in which division is always possible (except by zero). Every integral domain is contained as a subring in a field; the simplest example of this is the integral domain of ordinary integers which is contained in the field of rational numbers. Questions of unique factorization in integral domains are discussed and these ideas are then applied to polynomial rings and fields of rational functions. Incidentally, the characteristic of a ring is so defined that it is infinite (instead of zero) for a non-modular ring. It seems to the reviewer unfortunate to depart from common usage here, especially as the usual definition of the characteristic is just as natural as the one adopted in this book. The reviewer would also like to quarrel (in a mild way) with the author over the fact that he uses the German word "centrum" when the English word "central" has been used for this purpose for a long time.

2. One of the best examples of a non-commutative ring is the set of all square matrices of given order with coefficients in a given field. The author first studies rectangular matrices with coefficients in an arbitrary field, the results and methods being reasonably orthodox. Application is made to linear equations. Next he goes on to the study of square matrices and develops the theory of the minimum and characteristic equations, of elementary divisors, of similarity, and so on. The treatment of the minimum equation and related matters is a fine example of the elementary algebra in a non-commutative ring.

So far the results have been quite independent of the field in which the elements of the matrices lie. In the discussion of symmetric matrices (which are equivalent to quadratic forms) as well as skew, Hermitian, and other matrices, it is necessary to demand that the characteristic of the field be different from two. The author treats all these cases together in an elegant manner by introducing a general relationship that he calls an *involution* and which includes the various familiar types of symmetry

as special cases. The usual questions of equivalence, canonical forms, and so on, are taken up in this general case.

In a later chapter (10) the author returns to the subject of algebras of matrices, as the most important examples of linear associative algebras; furthermore, every linear associative algebra with unit element is isomorphic to a matrix algebra. Direct sums and products of matrix algebras are discussed, as well as the characteristic and minimum equations of a matrix algebra. The important concept of scalar extension is introduced and quadrate (normal simple) algebras are briefly treated, as are the important quadrate algebras known as cyclic algebras. Finally, the algebra consisting of all polynomials in a fixed matrix with coefficients in a given field is studied.

3. As much of the theory of finite groups as is essential for the theory of Galois is presented in Chapter 6. Thus we find here a discussion of quotient groups, composition series, an introduction to groups of permutations, and so on. The theorem of primary importance to an abstract treatment of group theory is that every finite abstract group is isomorphic to a group of permutations, and this theorem is included here. However, the determination of all representations of a given abstract group by transitive groups of permutations is not given, although this is of importance in the theory of resolvents as well as in the abstract theory of groups.

In Chapter 7 the author discusses algebraic extensions. First he treats the adjunction to a fundamental field $\mathfrak{F}$ of members of a larger field which is given in advance. Then he proves the existence and uniqueness of the root field of a polynomial with coefficients in $\mathfrak{F}$. This is a field $\mathfrak{K}$ containing a subfield isomorphic to $\mathfrak{F}$ such that the given polynomial factors into linear factors in $\mathfrak{K}$; furthermore it is the "smallest" such field. The simplest example is the field of complex numbers $\mathfrak{C}$, which is the root field of x^2+1 over the field of real numbers and whose construction in elementary algebra is a sample of the construction in the general case. That $\mathfrak{C}$ itself is then the root field over $\mathfrak{C}$ of *any* polynomial with complex coefficients is, of course, the fundamental theorem of algebra. This latter is stated but not proved in this text.

This chapter also contains a discussion of separable fields and of finite fields, the so-called *Galois* fields.

In Chapter 8 the Galois theory is given, based on the study of the group of automorphisms of a normal field. The Galois group of an equation follows as a consequence. Application is then made to the classical problem of the solution of equations by radicals. It is proved that the general equation of degree greater than four is not solvable by radicals, but the existence of special equations not solvable by radicals with respect to, say, the field of rational numbers is not discussed.

In this connection the reviewer would like to see some one break with tradition and present the general theory of the reduction of the solution of an equation to auxiliary equations, instead of treating merely the question of solvability by radicals. Hölder proved his theorem on composition series in order to be able to handle the general case, and the importance of the binomial equation is perhaps more historical than actual.

A *metacyclic* field is one for which the auxiliary equations just referred to are all cyclic. The next chapter (9) in this book contributes to the discussion of this case by giving a detailed account of the structure of fields that are cyclic with respect to a given field $\mathfrak{F}$. Various cases have to be distinguished, depending on the characteristic of $\mathfrak{F}$ and the presence of certain roots of unity. Incidentally, it is found that when $\mathfrak{F}$ is modular the simplest defining equations can *not* always be taken as binomial.

4. The ideas that are grouped around the theory of valuations are of great importance in modern algebra and arithmetic. The last two chapters in this book give

a good introduction to this subject. A field $\mathfrak{F}$ has a valuation ϕ if $\phi(a)$ is a real-valued function defined for every element a of $\mathfrak{F}$ with the properties of an absolute value:

$$\phi(0)=0, \quad \phi(a)>0 \text{ if } a\neq 0, \quad \phi(ab)=\phi(a)\phi(b), \quad \phi(a+b)\leqq\phi(a)+\phi(b).$$

A field $\mathfrak{F}$ is called complete with respect to ϕ if every sequence satisfying the Cauchy criterion has a limit. If $\mathfrak{F}$ is not complete, a *derived* field can be constructed that contains a subfield $\mathfrak{F}_0$ isomorphic to $\mathfrak{F}$ and has a valuation which for elements of $\mathfrak{F}_0$ is ϕ, and which is complete with respect to this valuation. The simplest example is the field of rational numbers $\mathfrak{R}$ with ϕ the ordinary absolute value; the derived field is then the field of real numbers. The so-called p-adic valuations of $\mathfrak{R}$ lead to the corresponding p-adic derived fields that were introduced by Hensel. The latter are examples of non-archimedean valuations and these are particularly important in arithmetic.

After studying the general theory of derived fields, the author attacks the problem of determining when a field $\mathfrak{F}$ has valuations and, when it has, of finding all the possible ones. For archimedean valuations it turns out that $\mathfrak{F}$ must be equivalent to a field of complex numbers and the valuation is equivalent to the ordinary absolute value. For non-archimedean valuations the author restricts himself to algebraic number fields of finite degree over $\mathfrak{R}$ and determines them all. Thus for $\mathfrak{R}$ itself the only non-archimedean valuations are the p-adic ones already mentioned; essentially the only archimedean valuation is the ordinary absolute value.

The subjects of algebra and its close relative, arithmetic, are very much alive now, and this book is a welcome addition to the literature. Besides furnishing an excellent introduction to abstract algebra, it contains several especially valuable features, such as the chapters on matrices and matrix algebras, the chapter on cyclic fields, and the chapters on valuations.

The only thing to be criticized in the book as a whole is, in the opinion of the reviewer, the fact that the fundamental character of the concept of the ideal is not emphasized. Ideals are not introduced until Chapter 11 (they are used there in the construction of derived fields) and then their fundamental importance is not made clear. Thus the related concept of homomorphism between rings is not even mentioned. The fact that the difference ring of a commutative ring with respect to an ideal is homomorphic to the given ring and that all homomorphisms can be so obtained is of primary importance in an abstract treatment of algebra. The construction of the root field (Chapter 7), the derived field (Chapter 11), and, of course, the rings arising from congruences with respect to a fixed modulus in elementary number theory and elsewhere, are all examples of this.

The book contains a great many problems, many of them simple illustrations of the body of the text. Others are more substantial additions to the subject matter. These problems are especially numerous and valuable in the chapters on the Galois theory. At the end of the book there is a glossary in which the terms most frequently used in the book are briefly defined; this should be very useful to the beginner.

A few minor slips and misprints were noted:

Page 24, line 9 from the bottom: should be a_n, b_n instead of a, b.

Page 25, line 5 from the bottom: should be $x-c$ instead of $x-a$.

Page 76, heading at top of page should be "Chapter IV."

Page 90, line 12 from the top: should be $j\neq t$.

Page 256, line 10 from the top should read "for every non-negative integer n."

Page 279, first line should read "Hence the limit is unity as stated in our lemma."

H. W. BRINKMANN

HILBERT AND ACKERMANN ON LOGIC

Grundzüge der theoretischen Logik. By D. Hilbert and W. Ackermann. 2d edition. Berlin, Springer, 1938. 8+133 pp.

This book gives an account of the symbolic treatment of certain topics of logic which have been particularly susceptible to that form of treatment. The first chapter treats of the propositional calculus. The second chapter gives a brief introduction to the calculus of classes, and shows how it can be made to give the traditional Aristotelian figures. The third chapter is devoted to the simple function calculus, that is, the calculus of classes of individuals and of dual and multiple relations between individuals. The fourth chapter elucidates the logical paradoxes and the use of a theory of types to avoid paradox.

The principal changes from the first edition are these: The nomenclature has been revised to agree with that of Hilbert-Bernays. The book has been brought up to date by rewriting the sections which deal with topics in which significant advances have been made in the last ten years. The deduction rules of the simple function calculus have been revised so as to eliminate certain inaccuracies occurring in the first edition.

This leaves the first and second chapters essentially unchanged. In the third chapter is added a proof of the independence of the axioms of the simple function calculus, and Gödel's proof that these same axioms are complete in one sense. The section on the decision problem of the simple function calculus is completely rewritten, reference being made to Church's result that any decision method that could be found for the simple function calculus would have to be non-constructive in a sense defined by Church and accepted by several other writers. The last chapter is considerably improved by the use of a simpler theory of types.

The first chapter opens with an intuitive discussion of the propositional calculus. This includes such topics as normal forms of a proposition, definition of universally true proposition, and principle of duality. The authors then introduce formal postulates for the propositional calculus and illustrate the use of the postulates by proving various of the intuitively obvious facts which they have been taking for granted in the previous intuitive discussion. This of course raises the question whether every universally true proposition is a consequence of the postulates, and also the question whether every consequence of the postulates is a universally true proposition. These are answered in the affirmative in the course of proving the completeness and consistency of the postulates. The consistency and independence of the postulates are proved by the familiar method of using numerical examples. Two kinds of completeness are proved, one kind which says that all universally true propositions are consequences of the postulates, and a sharper kind which says that if any proposition which is not a consequence of the postulates be adjoined as an additional postulate, then a contradiction will result. The decision problem for the propositional calculus is also solved. The decision problem is the problem of deciding whether a given proposition is universally true or not, and the solution formed part of the intuitive discussion which preceded the introduction of the postulates.

The results of the second chapter are not used elsewhere in the book, and so the second chapter would be irrelevant for anyone not acquainted with traditional logic. However, the chapter is short, and should interest enough readers to justify amply its inclusion.

The third chapter opens with a short intuitive discussion of the simple function

calculus, including, among other things, a definition of universally true proposition of the simple function calculus. Then formal postulates for the simple function calculus are introduced. These postulates are applied to the proof of a considerable number of propositions, many of which would be trivially true from purely intuitive considerations and all of which are universally true. However, the authors do not prove that every consequence of the axioms is universally true. They point out this omission and state that such a proof, if possible, would be of the highest importance. They do, however, prove the consistency of the postulates in the sense that there is no formula A such that A and not-A are both consequences of the postulates. This and the independence of the postulates are both proved by the use of numerical examples. They reproduce Gödel's proof that all universally true propositions are consequences of the axioms, thus proving completeness in one of the two senses mentioned previously. They also exhibit a formula for which they prove that it is not a consequence of the axioms and that it can be added to the axioms without getting an inconsistency, thus disproving completeness in the other sense mentioned previously. The section on the decision problem for the simple function calculus contains a general discussion and a summary (with references) of known results.

The fourth chapter, on the extended function calculus, does not contain a set of formal postulates, but is simply an intuitive discussion throughout. The application of this calculus to cardinal numbers and point set theory is sketched briefly. The various logical paradoxes which can arise in this calculus are discussed. A theory of types which avoids the known paradoxes is then explained. To show that this theory of types does not weaken the extended function calculus too much for the purposes of mathematics, several problems in the foundation of the theory of real numbers are handled by means of it.

The revision has increased the recognized value of this book as a reference work without sacrificing the quality of easy readability which distinguished the first edition. In fact, for persons with no previous knowledge of formal logic but with a reading knowledge of German, it offers a very satisfactory introduction to the subject.

BARKLEY ROSSER

DOETSCH ON THE LAPLACE TRANSFORMATION

Theorie und Anwendung der Laplace-Transformation. By G. Doetsch. (Grundlehren der Mathematischen Wissenschaften in Einzeldarstellungen, vol. 47.) Berlin, Springer, 1937. 8+436 pp.

A function $F(t)$, $(t \geqq 0)$, is defined here as an L-function if it is bounded in every finite interval in $t>0$, is absolutely integrable in $0 \leqq t \leqq T$ for some $T>0$, and has a convergent Laplace integral $\int_0^\infty e^{-st}F(t)dt$ for some real or complex $s=s_0$. The integrals are taken in the Riemann sense. After it has been proved that this integral then converges for every s in the half-plane $R(s)>R(s_0)$, the class of l-functions is defined as the aggregate of all functions $f(s)$ which are Laplace transforms of L-functions: $f(s)=\int_0^\infty e^{-st}F(t)dt=\mathfrak{L}\{F\}$. The principal aims of the book are to show how the character of and operations upon functions belonging to one of these classes are reflected upon the corresponding functions of the other, and to demonstrate a wide variety of applications of these results. Functions of other classes and other transformations play only minor rôles here, except for the two-sided Laplace transformation and the Mellin transformation, the study of which is made to depend in a simple way upon that of $\mathfrak{L}\{F\}$. After discussing the advantages of using Lebesgue integrals, limit in the mean, and other concepts in modern analysis, the author presents his reasons for using only the older concepts (Chapter 3, §1; 8). Aside from his intention to adapt the book to readers without such modern equipment, he points out that in the applications the local limit rather than the limit in the mean is needed, and that functions appear there which are L-functions but do not belong to the Lebesgue square integrable class. The Laplace transformation of L-functions into l-functions is not a continuous transformation, so difficulties are encountered here which are not present when the functions belong to Hilbert space.

The book is divided into five parts covering, in order, the general theory, its use in finding expansions of certain functions in series, the asymptotic behavior of functions, integral equations, and differential equations. The applications are said to begin with Part II, but much of Part III is again general theory, so less than half the book is devoted to the applications. The appendix contains a useful table of transforms of functions together with their half-planes of convergence.

Part I first presents a few fundamental concepts of function spaces and transformations. The short historical account of the genesis of the Laplace transformation which follows is supplemented later in Chapter 6 by a section on the early history of the Fourier transform, and again by remarks on the more recent sources among the many reference notes in the appendix. The chapters on general theory give an extensive treatment of half-planes of convergence, the regions of regularity, and analytic continuation of l-functions. It is important for the reader to note that $F(t)$ is assumed to be an L-function when the contrary is not stated; otherwise, several theorems in Chapters 4 and 6 will be confusing. A good account of inversion formulas follows; this includes the complex inversion integral and the formulas of Post, a little-known one by Phragmén, series inversion formulas, and others. The final chapter of this part, on the mapping of the fundamental operations, determines the operations in either the L- or l-space which correspond to linear substitution, integration, differentiation, or multiplication of functions in the other.

A single chapter, illustrating the mapping of series developments of particular functions, makes up Part II. Part III begins with Chapter 10, containing thirteen

abelian theorems, in which the asymptotic behavior of the transform $f(s)$ is determined from the behavior of $F(t)$, and eight theorems of the opposite, or Tauberian, type. These theorems, taken from a variety of sources, are well organized here both with respect to their proofs and with respect to the points at which asymptotic behavior is determined. They are applied in the remaining four chapters of Part III to obtain asymptotic series expansions of the functions $f(s)$ and $F(t)$. Applications of these expansions include asymptotic developments of $\log \Gamma(s)$, $J_0(x)$, $\operatorname{erf}(s)$, and a derivation of the prime number theorem.

The earlier theory is used in Part IV to solve the Volterra integral equation with kernel $K(t-\tau)$, (Faltung type). This is followed by a discussion of the more general non-linear integral equation of this type and an interesting treatment of fractional integration and differentiation. The inverse of the problem of integral equations, that of determining transcendental addition theorems, is also illustrated.

Part V, on the solution of differential equations by the Laplace transformation, contains a detailed treatment on ordinary differential equations followed by solutions of boundary problems in partial differential equations of parabolic, hyperbolic, and elliptic types. Some observations from Picone's study of the compatibility of boundary conditions are included to real advantage in the treatment of partial differential equations. Space was not available here to do full justice to the problems of inversion and establishing the solution, of course, and it is to be hoped that the reader will not conclude that these steps are possible only in a limited number of problems. In the final chapters the deficiencies of the Heaviside operational method are made clear, and the principles of Huygens and Euler as applied to boundary problems are illustrated.

The book is carefully written. The number of misprints in addition to those listed at the beginning is very small. It is fortunate that the work of selecting this material from the sources in several fields and giving a unified account of it for the first time has been done so accurately. Nearly all the results are given as theorems; this does much to compensate for the crowded typography of the book. The arrangement of the references is rather distracting; the reader is too frequently asked to turn to other parts of the book. The analogies with the theory of power series and the applications to Dirichlet series which are prominent throughout the theoretical part contribute much to an interesting development.

R. V. CHURCHILL

SHORTER NOTICES

Lehrbuch der Gruppentheorie. Vol. 1. By Hans Zassenhaus. (Hamburger Mathematische Einzelschriften, no. 21.) Leipzig and Berlin, Teubner, 1937. 6+151 pp.

This text, mostly on finite discrete groups, was suggested, the author states, by Artin's Hamburg lectures of 1933–1934. Its spirit is that of van der Waerden's *Moderne Algebra*, but of course this specialized book covers much more ground on groups than does that general treatise on modern algebra. Readers familiar with older books on finite groups, or with papers written in the classical manner, will recognize much of the material; the technique of the proofs is frequently simpler, less fortuitous, and more direct than the old. A consistent attempt is made to exploit the principle of homomorphic mapping to the limit. In addition to the practitioners of this technique cited by the author, particularly E. Noether and her many pupils in abstract algebra since about 1921, Gauss should be remembered as its originator. The abstract notions of congruence relations and equivalence relations are seen to have far wider scope than those for which they were devised in the theory of numbers. Dedekind should also be remembered in this connection.

The five chapters of the book present a wealth of material in compact form. The insistence is upon general theorems; the special properties of substitution groups, for instance, receive only passing mention, and linear groups are not discussed. A considerable amount of quite recent material is included; thus there is some account of P. Hall's work on groups whose order is a power of a prime, and Schreier's refinement of the Jordan-Hölder theorem is proved. Specific citations to the literature are likewise recent, and contain references to Ore's work on structures and group theory and G. Birkhoff's on transitive groups. In the historical references, G. A. Miller's reports on certain aspects of group theory and the account in his *Collected Works* might have been included with advantage. As would be expected, most of the references are to modern German work.

As the exposition itself is condensed, we can give only a brief indication of the contents. After a somewhat crowded first chapter of 27 pages dealing with fundamental notions, including the algebra of complexes, the book gets into its stride in the second chapter, devoted to homomorphism (as applied to groups), and groups with operators. This chapter of 45 pages includes, incidentally, the automorphisms of a group, commutative groups, and normal series, also a brief exposition of the groups occurring in algebra. Ideals are here introduced in connection with the subgroups of a skew ring.

One of the author's aims is the construction in a finite number of non-tentative steps of the several algebraic objects of the theory. Accordingly, Chapter 3, of 23 pages, is concerned with the construction of composite groups. The basis theorem for Abelian groups is quickly derived after a brief discussion of direct products (following Fitting, 1934) and some necessary preliminaries on matrices. The chapter concludes with an account of Schreier's Erweiterungstheorie and applications.

The fourth chapter presents Sylow's theorem, its extensions and consequences; and p-groups (groups of prime-power order). It includes a short discussion of Hamiltonian groups. Among the examples is P. Hall's generalized Sylow theorem for finite solvable groups. The concluding chapter, of 15 pages, presents, among other things, monomial representations, the theorems of Grün (1935), groups with all Sylow sub-

groups cyclic, and the principal-ideal theorem of class-field theory transposed to group theory. Throughout the book there are short sets of approachable exercises.

The treatment here presented achieves a certain unity which the classical presentation lacked. It also exhibits the theory of abstract finite groups (infinite abstract groups are not discussed in much detail) against the background of modern algebra as that subject has developed in the past seventeen years. In the reviewer's opinion, this method of approach is likely to appear more coherent than the former to students approaching groups in detail for the first time. A considerable number of new technical terms must be kept in mind, but with either van der Waerden's book or Albert's as a preliminary course, the terminology and the ideas behind it will offer no serious difficulty.

E. T. BELL

Axiomatische Untersuchungen zur projektiven, affinen und metrischen Geometrie. By Eugen Roth. (Forschungen zur Logik und zur Grundlagen der exacten Wissenschaften, new series, no. 2.) Leipzig, Hirzel, 1937. 58 pp.

This work, incorporating the material of a doctoral dissertation, falls into three main subdivisions: (1) an axiomatic development of projective, affine, and euclidean geometry: (2) concerning the axiomatizing of geometries; (3) the monomorphism of the postulate systems of projective, affine, and euclidean geometry. The author uses formal logical symbols explained in a glossary, whose introduction renders the reading difficult without considerable study. The emphasis here is upon the logical structure, and because of this point of view certain problems are brought out which were largely ignored by earlier writers on postulates. The specializations required to obtain affine geometry from projective, and euclidean from affine, are shown to be postulational, not definitional (American readers may recall R. L. Moore's critical remarks on this score in reviewing Veblen and Young's *Projective Geometry*). The "simplifications" familiar in reducing postulates for projective geometry, by prescribing lines and planes to be sets of points, rather than wholly undefined symbols, do violence to the principle of duality, as this writer points out effectively. The author remarks that the complete postulates for projective geometry require continuity of points on a line, and hence the "incidence geometries" illustrated by finite geometries are not "projective." Critical remarks upon Hilbert's "completeness postulate" for euclidean geometry (as given prior to the 1930 edition, see 4th to 6th editions) incorporate much that is found in recent German literature on the subject. The author is apparently unaware of the work of H. G. Forder, *The Foundations of Euclidean Geometry*. He is aiming at certain logically vulnerable spots in modern axiomatic treatments, rather than giving a survey of the subject as a whole.

A. A. BENNETT

NOTES

The Fifth International Congress for Applied Mechanics is to be held during the week following the Semicentennial meeting of the Society in New York. It is scheduled for September 12–16, 1938, at Harvard University and the Massachusetts Institute of Technology. It is expected that nearly one hundred foreign scientists will be in attendance in addition to a large group from American institutions. Rooms will be furnished in the residence halls at Harvard University at one dollar per person per night. The Congress fee is five dollars. The work of the Congress will be accomplished by means of technical sessions at which communications from members will be read and discussed. The field of Applied Mechanics will be divided into the following general heads: I. Structures, Elasticity, Plasticity, Fatigue, Strength Theory, Crystal Structure. II. Hydro- and Aerodynamics, Gasdynamics, Hydraulics, Meteorology, Water Waves, Heat Transfer. III. Dynamics of Solids, Vibration and Sound, Friction and Lubrication, Wear and Seizure. There will be six general lectures given by invitation, a symposium on Turbulence, and about one hundred communications from members. Members of the American Mathematical Society will be welcome at the Congress. Details concerning the program and arrangements can be obtained by writing to the Secretary, Professor J. C. Hunsaker, Massachusetts Institute of Technology, Cambridge, Massachusetts.

A new mathematical journal is being published in Portugal; it is called Portugaliae Mathematica. The first number of the first volume appeared in May, 1938.

Volume 22 of the American Mathematical Society Colloquium Publications, *Summable Series and Convergence Factors*, by C. N. Moore, was released early in June, 1938. The list price is $2.00. Members of the American Mathematical Society receive 25 per cent discount.

Work is being done on the reconstruction of the mathematical and philosophical thought of Pythagoras. Original contributions on this subject are invited. All communications should be addressed to Mr. Carolus Ferrari, P. O. B. 250, Grand Central Annex, New York City.

The papers and letters of the late Dr. Elihu Thomson have been deposited with the American Philosophical Society. The Society hopes to add to the collection to make it as complete as possible, and will be glad to receive pieces of Thomsoniana or information regarding them. Material should be sent to the American Philosophical Society, Philadelphia, Pa.

The James Alfred Ewing Medal was awarded to Mr. C. S. Franklin for meritorious contributions to the science of engineering in the field of research.

Awards in the William Lowell Putnam Mathematical Competition are announced by the Mathematical Association of America. The first prize of $500 was won by the University of Toronto, the second prize of $300 was awarded to the University of California at Berkeley, and the third prize of $200 was awarded to Columbia University. A. J. Coleman, I. Kaplansky, and N. S. Mendelsohn were the members of the winning team. S. P. Frankel, W. M. Kincaid, and C. W. Lippman represented the University of California; and S. W. Benson, B. A. Jacobson, and Joseph Stalsinger comprised the team from Columbia University. The William Lowell Putnam Prize

Scholarship has been awarded by the Division of Mathematics, Harvard University, to Mr. I. Kaplansky. He plans to use the award during the academic year 1939–1940.

Professor W. Sierpiński, of the University of Warsaw, has been reelected president of the Society of Science of Warsaw. Professor S. Mazurkiewicz, also of the Univeristy of Warsaw, has been elected general secretary.

Dr. K. Borsuk and Dr. B. Knaster, of the University of Warsaw, have been elected corresponding members of the Society of Science of Warsaw.

On May 20, 1938, at the dedication of the Franklin Institute of Philadelphia, the University of Pennsylvania conferred an honorary degree upon Dean G. D. Birkhoff, of Harvard University.

Professor J. W. Lasley, Jr., of the University of North Carolina, has been elected president of the North Carolina Academy of Science, for the year 1938.

Professor Solomon Lefschetz, of Princeton University, has been invited to address the British Association for the Advancement of Science at its August, 1938, meeting. His topic will be *Fixed points of transformations.*

Professor M. H. Stone, of Harvard University, has been elected a member of the National Academy of Sciences.

At the annual meeting of the American Academy of Arts and Sciences, held on May 11 in Boston, Dean G. D. Birkhoff spoke on *Mathematical advances since* 1900 *and their influence on scientific thought.*

A Guggenheim Fellowship has been awarded to Assistant Professor D. H. Lehmer, of Lehigh University. Professor Lehmer will work at various English universities on the analytic theory of numbers.

Dr. B. J. Pettis, of the University of Virginia, has been appointed Sterling Fellow at Yale University.

Dr. Felix Burkhardt, of the Univeristy of Leipzig, has been promoted to an assistant professorship.

Dr. Alfred Kneschke, of the Technical School in Dresden, has been promoted to an assistant professorship.

Dr. W. Nikliborc, of the University of Lwow, has been appointed to a professorship at the Warsaw Technical School.

Dr. W. Orlicz, of the University of Lwow, has been appointed to a professorship at the University of Poznan.

Dr. G. E. Uhlenbeck, of the University of Utrecht, has been appointed visiting professor of theoretical physics at Columbia University.

Dr. Bernard Cannon, of the University of Liverpool, and Mr. J. S. de Wet, of the University of Cape Town, have been awarded Commonwealth Fund Fellowships, and will pursue studies in mathematics at Princeton University and the Institute for Advanced Study, respectively.

Assistant Professor R. P. Agnew, of Cornell University, has been promoted to a professorship.

Dr. Theodore Bennett, of Marietta College, has been promoted to an assistant professorship.

Dr. M. T. Bird, of Utah State Agricultural College, has been promoted to an assistant professorship.

Mr. Garrett Birkhoff, of Harvard University, has been promoted to an assistant professorship.

Dr. Julia W. Bower, of Connecticut College, has been promoted to an assistant professorship.

Professor Evelyn Carroll-Rusk has been appointed dean at Wells College.

Assistant Professor J. W. Cell, of North Carolina State College, has been appointed to an associate professorship.

Assistant Professor R. V. Churchill, of the University of Michigan, has been appointed to an associate professorship.

Associate Professor R. D. Douglass, of the Massachusetts Institute of Technology, has been promoted to a professorship.

Professor W. H. Durfee has been made dean of Hobart College.

Dr. A. L. Foster, of the University of California at Berkeley, has been promoted to an assistant professorship.

Professor J. W. Glover, of the University of Michigan, has retired with the title emeritus professor.

Mr. J. W. Green, of the University of California at Berkeley, has been appointed Benjamin Peirce instructor in mathematics at Harvard University.

Dr. D. W. Hall has been appointed National Research Fellow for 1938–1939. He will study at the University of Pennsylvania.

Dr. G. G. Harvey, of the Massachusetts Institute of Technology, has been promoted to an assistant professorship of physics.

Associate Professor Frank Irwin, of the University of California at Berkeley, has been given the title emeritus.

Dr. H. J. Kersten, of the University of Cincinnati, has been promoted to an assistant professorship.

Dr. B. F. Kimball has been appointed engineering mathematician, Depreciation Unit of the Division of Research and Valuation of the State Public Service Commission, New York City.

Assistant Professor J. H. Kusner, of the University of Florida, has been promoted to an associate professorship.

Dr. H. M. MacNeille, of Harvard University, has been appointed associate professor of mathematics at Kenyon College.

Dr. W. T. Martin, of the Massachusetts Institute of Technology, has been promoted to an assistant professorship.

Professor Karl Menger, of the University of Notre Dame, will be visiting professor during the summer quarter at the University of California at Berkeley.

Dr. F. J. Murray, of Columbia University, has been promoted to an assistant professorship.

Mr. D. T. Perkins, of Yale University, has been appointed Benjamin Peirce instructor in mathematics at Harvard University.

Assistant Professor W. S. Schlauch, of New York University, has been promoted to an associate professorship.

Dr. Ruth G. Simond, of the University of Michigan, has been appointed to an assistant professorship at Hampton Institute.

Dr. Joseph Slepian has been appointed associate director of the Westinghouse Research Laboratories at Pittsburgh.

Professor C. H. Smiley has been made chairman of the department of astronomy and director of the Ladd Observatory at Brown University.

Professor Virgil Snyder, of Cornell University, has retired with the title professor emeritus.

Assistant Professor A. W. Tucker, of Princeton University, has been promoted to an associate professorship.

Mr. C. B. Tucker, of Kansas State Teachers College, has been promoted to an assistant professorship.

Dr. R. J. Walker, of Cornell University, has been promoted to an assistant professorship.

Associate Professor W. M. Whyburn, of the University of California at Los Angeles, has been promoted to a professorship.

Professor E. P. Wigner, of the University of Wisconsin, has been appointed professor of mathematical physics at Princeton University.

The following appointments to instructorships have been announced: Bryn Mawr College: Mr. M. P. Fobes; University of California at Los Angeles: Dr. A. E. Taylor; Case School of Applied Science: Dr. J. M. Dobbie; Creighton University: Mr. J. D. Fitzpatrick; Harvard University: Dr. L. A. Pipes; Haverford College: Dr. C. B. Allendoerfer; University of Illinois: Dr. P. T. Maker, Dr. E. L. Welker; University of Michigan: Dr. C. J. Nesbitt; Middlebury College: Mr. L. B. Hedge; North Park College (Chicago): Mr. C. G. Erickson; University of Notre Dame: Dr. P. M. Pepper; Pennsylvania State College: Dr. C. H. Graves; Princeton University: Mr. W. C. Strodt; Purdue University: Dr. D. R. Shreve, Dr. M. S. Webster; Wellesley College: Miss Melita A. Holly; Wesleyan University: Dr. J. W. Wrench; Yale University: Dr. D. C. Murdoch.

Professor Carl Gundersen, of Oklahoma Agricultural and Mechanical College, died on April 11, 1938. He had been a member of the American Mathematical Society since 1902.

Dr. G. E. Hale, who was until 1923 director of the Mount Wilson Observatory, died in Pasadena on February 21, 1938.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

287. C. R. Adams and J. A. Clarkson: *The type of certain Borel sets in several Banach spaces.*

Let AC represent the class of functions $x(t)$ absolutely continuous on $0 \leqq t \leqq 1$, CBV the class of continuous functions of bounded variation, C the class of continuous functions, R the class of properly Riemann integrable functions, and R^* the class of properly or improperly Riemann integrable functions. Oxtoby (see this Bulletin, vol. 43 (1937), pp. 245–248) has shown that the subsets C, R, and R^* of each space $L_p([0, 1])$, $(p \geqq 1)$, are $F_{\sigma\delta}$ sets of first category. In the present paper, the determination of their Borel type is completed by showing that each is no $G_{\delta\sigma}$. It is proved also that the subset AC of each of the spaces C, L_∞ (or M), and L_p is likewise an $F_{\sigma\delta}$ but no $G_{\delta\sigma}$, and that $CBV \subset L_p$ is both an $F_{\sigma\delta}$ and a $G_{\delta\sigma}$ but neither an F_σ nor a G_δ. The chief tool employed is a characterization of a G_δ in any metric space which runs roughly as follows: E is a G_δ if and only if no sequence $\{x_n\} \subset E$ which converges *rapidly enough* tends to a limit outside of E. For resolving questions of the sort here considered, the range of applicability of the methods mainly employed is by no means restricted to Banach spaces. (Received May 4, 1938.)

288. G. E. Albert: *Asymptotic forms for the generalized Legendre functions.*

Complete asymptotic forms for the functions $P_n^m(z)$ and $Q_n^m(z)$, n, m, and z complex, are derived under the following sets of conditions: (1) n is large, m is relatively moderate, and (a) z is exterior to some small neighborhoods (depending on n) of the critical points $z = \pm 1$, or (b) z is within such a neighborhood; (2) m is large, n is relatively moderate, and (a) $|z| < c|m|$, or (b) $|z| \geqq c|m|$, is valid for the smaller of the two indices in any bounded domain. The cases (1a) and (2a) are subject to Stokes' phenomenon; the discontinuities incurred are studied completely. The formulas developed corroborate all previously known results. The results for case (1b) are almost entirely new, only a few special cases of them having been studied before. Those for case (2b) are totally new. The method of the paper consists in an identification of the associated Legendre equation with a more general equation whose asymptotic solutions have been given by R. E. Langer in terms of elementary functions or Bessel functions. The interdependence of solutions leads to the calculation of the desired forms. (Received May 19, 1938.)

289. J. J. De Cicco: *Additional properties of the second derivative of a polygenic function.*

This is a continuation of the paper, *The second derivative of a polygenic function*, by Kasner (Transactions of this Society, vol. 30 (1928), pp. 803–818). Kasner has shown that the second derivative $\sigma = d^2w/dz^2$, taken over an arbitrary path of approach to the *fixed* point z, defines a point transformation Σ between the centers of curvature $Z = X + iY$ relative to the fixed point $z = x + iy$, of the Z-plane, and the points of the σ-plane. Kasner proved that there is a fundamental cardioid E in the σ-plane such that the correspondence between the points in the Z- and σ-planes is (1, 1) inside E, (2, 1) on E, and (3, 1) outside E. The ∞^1 lines through z are converted into the ∞^1 lines tangent to E. The present paper shows that the ∞^2 straight lines not through z are converted into a congruence Γ of limaçons. In Γ there is a unique circle C_M. This corresponds to two lines L_C and $L_{C'}$. The correspondence between all the other lines and limaçons of Γ is (1, 1). In Γ there are ∞^1 cardioids G. These have concentric base circles and their poles are on E. The minimum cardioid is E, and there is one maximum cardioid M. Its base circle is C_M. If L_E and L_M are the lines which are transformed into E and M, then L_E, L_M, L_C, $L_{C'}$ are concurrent, and L_E, L_M separate L_C, $L_{C'}$ harmonically. Finally, the ∞^1 lines which are transformed by Σ into the ∞^1 cardioids G envelop a conic. (Received May 16, 1938.)

290. J. J. De Cicco: *Asymptotic direction of a field of lineal elements.*

This is a continuation of the paper, *Geometry of turbines, flat fields, and differential equations*, by Kasner and the present author, and abstract 43-5-260 by the author, *The differential geometry of series of lineal elements*. Let F be an element in the field adjacent to a given element E, and let ER be the turbine of intersection of the tangent flat fields at E and F. The limiting turbine directions of EF and ER are called *reciprocal* at E. Theorem 1: Let T_0 be the principal tangent turbine (the extremal general curvature direction), and let T and T' be two tangent turbines at E. Let S and S' be the distances of T and T' from T_0. A necessary and sufficient condition that the directions T and T' be reciprocal is $SS' = -K_0/\lambda$, where K_0 is the extremal general curvature and λ is the equi-parallel curvature. Self-reciprocal directions are called asymptotic directions. Theorem 2: For a series to be an asymptotic series, it is necessary and sufficient that its osculating flat fields coincide with the tangent flat fields. Theorem 3: A necessary and sufficient condition that a series be an asymptotic series is $\tau = \pm G^{1/2}$, where τ is the torsion of the series and G is the curvature of the field (the analogue of the Beltrami-Enneper theorem). (Received May 16, 1938.)

291. F. C. Gentry: *Cremona involutions determined by a pencil of quartic surfaces.*

A projective correspondence is established between the members of a pencil of quartic surfaces containing a double line d and the points of d. A line through a point of d meets the surface of the pencil corresponding to that point in two points which are in involution. The special configurations of the F-curves of the second kind of the transformation, which arise when the residual basis curve of the pencil of surfaces is composite, are determined. (Received May 23, 1938.)

292. F. C. Gentry: *Quaternary Cremona groups of ternary type.*

The webs of quartic surfaces of degree two which contain a double curve and one or more simple points in their bases are determined. Each such web of surfaces deter-

mines an involution in space. The involutions obtained by allowing the simple base-points of such a web of surfaces to vary while the remainder of the base is held fixed are used to generate groups of Cremona transformations. These groups are shown to be simply isomorphic to the linear groups $g_\rho(\alpha)$ or $g_\rho(r, \epsilon, e)$ discussed by Coble (*A class of linear groups with integral coefficients*, Duke Mathematical Journal, vol. 3 (1937), pp. 175–199). (Received May 23, 1938.)

293. E. R. Lorch: *Bicontinuous linear transformations in complex euclidean spaces.*

A complex euclidean space E is a unitary space, a Hilbert space, or a non-separable space of the Hilbert type. A bicontinuous transformation T in E is a bounded linear transformation which possesses a bounded inverse. Let $\{\phi_\alpha\}$ be a complete orthonormal set in E, α ranging over any preassigned set. Let $\{\psi_\alpha\}$ be a set of linearly independent vectors spanning E. (The vectors of a set are said to be linearly independent if, for any n vectors $f_1, \cdots, f_n$ in the set, the condition $\sum_{i=1}^{n} a_i f_i = 0$ with complex a_i implies $a_i = 0$, $(i = 1, \cdots, n)$.) A complete analysis is given of the internal structure of the set $\{\psi_\alpha\}$ in order that there exist a bicontinuous transformation T such that $T(\phi_\alpha) = \psi_\alpha$. Conditions necessary and sufficient for the existence of T are: (1) the boundedness of the sets of numbers $\{\|\psi_\alpha\|\}$ and $\{\|\psi_\alpha\|^{-1}\}$; (2) the uniform boundedness of the bounds of all projections P constructed as follows: the domain of P is the n-space spanned by any set of n vectors $\{\psi_{\alpha_1}, \cdots, \psi_{\alpha_n}\}$. The range of P is the space spanned by any subset of the set $\{\psi_{\alpha_1}, \cdots, \psi_{\alpha_n}\}$. (Received May 6, 1938.)

294. A. P. Morse: *The behavior of a function on its critical set.*

Consider the statement: If R is an open subset of euclidean space of n dimensions and f is a function on R to the set of real numbers which is of class C^m, $(m \geqq 0, n \geqq 1)$, then f transforms its set of critical points into a set of (linear) measure zero. Whitney (Duke Mathematical Journal, vol. 1 (1935), p. 514) has shown the statement false if $m = n - 1 \geqq 1$; Marston Morse and Sard, in an unpublished paper, have shown on the other hand that the statement is true in case m is the greatest integer in $n + (n-3)^2/16$. The present paper answers in the affirmative the natural question: Is the statement true for $m = n$? This problem was suggested to the author by Marston Morse. (Received May 6, 1938.)

295. B. J. Pettis: *On differentiation in Banach spaces.*

Given an additive function $X(R)$ defined and of bounded variation from euclidean figures to an arbitrary Banach space X, a condition can be stated in terms of linear functionals which is sufficient that $X(R)$ be strongly differentiable a.e. (almost everywhere). From this theorem can be drawn the results of Clarkson (Transactions of this Society, vol. 40 (1936), p. 396), Dunford and Morse (ibid., vol. 40 (1936), p. 415), Gelfond (Comptes Rendus de l'Académie des Sciences de l'URSS, vol. 17), and the author (this Bulletin, vol. 44 (1938), pp. 420–428) concerning differentiation of such functions. The paper also includes proofs of the following theorems: (1) if an additive BV function $X(R)$ is weakly differentiable a.e., then it is strongly differentiable a.e.; and (2) if the space X has an equivalent uniformly convex norm, then X is regular. (Received May 25, 1938.)

296. Olga Taussky: *Differential equations and hypercomplex systems.*

An interesting class of partial differential equations arises in the following way from hypercomplex systems over the field of real numbers. Let S be a hypercomplex system with n base elements $e_1, \cdots, e_n$ and $x_1e_1+\cdots+x_ne_n$ a general element of S, where the x_i are any real numbers. The norm of $x_1e_1+\cdots+x_ne_n$, if defined by means of the regular representation of S, is a homogenous function $f(x_1, \cdots, x_n)$ of the nth degree in $x_1, \cdots, x_n$. If the coordinates are replaced by the differential operators $\partial/\partial x_1, \cdots, \partial/\partial x_n$, a differential operator $f(\partial/\partial x_1, \cdots, \partial/\partial x_n)$ is obtained. When S is the system of complex numbers or quaternions, the differential operators so obtained are the Laplace's operators $\partial^2/\partial x_1^2+\partial^2/\partial x_2^2$ and $\partial^2/\partial x_1^2+\partial^2/\partial x_2^2+\partial^2/\partial x_3^2+\partial^2/\partial x_4^2$, respectively. It is known that each of a pair of functions satisfying the Cauchy-Riemann equations satisfies the Laplace equation; similarly for each of a set of four functions satisfying the Dirac equations. Let $l_i=a_{kj}^{(i)}\,\partial u_k/\partial x_j$, $(i=1, \cdots, n)$, be n linear differential forms with constant coefficients such that $\partial^2 u_i/\partial x_1^2+\cdots+\partial^2 u_i/\partial x_n^2=a_{i1}\,\partial l_1/\partial x_{i_1}+\cdots+a_{in}\,\partial l_n/\partial x_{i_n}$, $(i=1, \cdots, n)$, where a_{ik} are constants and the i_k are any of the numbers $1, \cdots, n$. The numbers n for which such relations exist can be completely determined by properties of hypercomplex systems. (Received May 27, 1938.)

297. Jesse Douglas: *Green's function and the problem of Plateau.*

Previous solutions given by the author of the general topological form of the problem of Plateau were based on an explicit formula for the Green's function of the basic Riemann surface R in terms of θ-functions and abelian integrals. (See Journal of Mathematics and Physics, vol. 15 (1936), pp. 55–64 and 105–123; also a more comprehensive paper in the Annals of Mathematics (in press), and a note in the Proceedings of the National Academy of Sciences (August, 1938), the last two bearing the title *Minimal surfaces of higher topological structure*). The present paper utilizes Green's function in an intrinsic way, without employing for it any explicit formula. The analytic procedure is the same in all cases, regardless of the particular topological form of R. The central feature is an explicit construction of a special variation of R for the corresponding algebraic curve $\mathcal{A}$, under which Green's function undergoes a particular variation that directly relates the variational condition $\delta A(g, R)=0$ for the fundamental functional $A(g, R)$ to the condition for a minimal surface. A note presenting the main features of the paper will appear in the Proceedings of the National Academy of Sciences (August, 1938), and a detailed version in the Journal of Mathematics and Physics. (Received June 18, 1938.)

298. Jesse Douglas: *Minimal surfaces of higher topological structure.*

This paper first reviews and elaborates the essential features of the author's previous papers on the subject (Journal of Mathematics and Physics, vol. 10 (1931), pp. 310–359; ibid., vol. 15 (1936), pp. 55–64, 105–123; Transactions of this Society, vol. 33 (1931), pp. 263–321; ibid., vol. 34 (1932), pp. 731–756). It then provides various new results, relating particularly to the case where the prescribed genus of the minimal surface is greater than zero. An explicit formula in terms of θ-functions and abelian integrals is given for the Green's function of a general Riemann surface R, with any finite number of boundaries and of any finite genus. As was announced in

the last cited Journal publication (p. 108), the theory is extended to the case where the contours are completely general Jordan curves, not bounding any surface of finite area. As throughout the author's work on the Plateau problem, the conformal mapping of plane regions is included. A more detailed discussion is given of a certain inequality necessary for the application of the general theory to this particular case. A comprehensive presentation will appear in the Annals of Mathematics and the Proceedings of the National Academy of Sciences (August, 1938). (Received June 16, 1938.)

299. N. A. Hall: *The number of representations function for binary quadratic forms.*

Explicit expressions are given for the number of representations of any positive integer in any positive definite binary quadratic form of discriminant such that there is a single class in each genus. To apply the Dirichlet formula for the number of representations for integers prime to the discriminant, the reduction formulas of G. Pall (Mathematische Zeitschrift, vol. 36 (1932), pp. 321–343) for the representations of integers not prime to the discriminant are extended. These show that aside from a few trivial exceptions specially treated, when the discriminant contains no odd prime square factors, the occurrence of factors of the discriminant in the number to be represented affects only the representations or non-representation; and in case it is represented, does not affect the number of representations. The representability is a function of the power of the prime factors of the discriminant and the characters for the particular form involved. This is given explicitly according to a convenient congruential classification of discriminants. When the discriminant contains odd prime square factors, use is made of theorems previously obtained by the author (cf. this Bulletin, vol. 43 (1937), p. 202) showing that only ten such cases occur when there is a single class of forms to a genus. This permits treatment of these cases individually and thus completes the solution. (Received June 14, 1938.)

300. E. R. Kolchin: *On complementary manifolds in certain Banach spaces.*

Let $\mathfrak{B}$ be a Banach space. Consider the question "Does every closed linear manifold in $\mathfrak{B}$ have a complementary manifold?" F. J. Murray has shown (Transactions of this Society, vol. 41 (1937), pp. 138–152) that the answer for $\mathfrak{B}=L_p$ or l_p, $(1<p\neq 2)$, is "no." In this note the arguments of Murray are modified to show that the answer for $\mathfrak{B}=c$, c_0, or m is "no." (Received June 3, 1938.)

301. Walter Leighton: *Sufficient conditions for the convergence of a continued fraction.*

Let (1) $1+K_1^{\infty}[a_n/1]$ be a continued fraction in which the quantities a_n are arbitrary complex numbers. Sets of sufficient conditions for convergence of (1), established by J. Worpitsky, E. B. Van Vleck, A. Pringsheim, O. Szász, H. S. Wall, W. Leighton, and others, have required that at least an infinite subsequence of the numbers a_n be less than 1/4 in absolute value. This condition can be removed as follows. If $|1+a_2|\geq 1+|a_1|$, $|a_{2n+1}|\leq m$, $|a_{2n}|\geq(2+m)/(1-m)$, $|a_{2n+2}|\geq 2+m+m|a_{2n}|$, $n=1, 2, 3, \cdots$, where m is any fixed positive number <1, the continued fraction (1) converges. A similar theorem with the roles of the "odd" and "even" a_n interchanged is also valid. In particular, the continued fraction with $a_n=-1/4-e_n$ can be shown to converge for a wide choice of the numbers $e_n>0$, even though if $e_1=e_2=e_3=\cdots=e$, (1) diverges (Szász) for all $e>0$. (Received June 4, 1938.)

302. B. A. Lengyel: *Bounded self-adjoint operators and the problem of moments.*

In this paper, a connection is established between the theory of moments and the theory of bounded self-adjoint operators. Let H be a bounded self-adjoint operator of the Hilbert space. The resolvent, R_z of H, can be represented by a Stieltjes integral with the denominator $z-\lambda$, if the problem of moments with $\mu_n=(H^n f, f)$ has a unique solution for all f's in the Hilbert space. The existence of a solution is immediately secured by the definite positive character of the quadratic forms $|\sum_{i=0}^{n}\alpha_i H^i f|^2=\sum_{i=0}^{n}\alpha_i\bar{\alpha}_j(H^i f, H^j f)$, $(n=1, 2, \cdots)$. The uniqueness of the solution is established through a lemma concerning the closed linear manifolds generated by $[Hf, H^2 f, H^3 f, \cdots]$. Since it is known that the spectral theorem follows readily from the Stieltjes integral representation of the resolvent, this paper can be considered as one of the numerous proofs of the spectral theorem. (Received June 20, 1938.)

303. Deane Montgomery and Leo Zippin: *Non-abelian compact connected transformation groups of three-space.*

In this paper, the authors conclude their study of *compact connected effective transformation groups of euclidean three-space* (hereafter, $Tg(E_3)$) with a final theory on the non-abelian case. The following result, already established by the authors, is used: The only *abelian* $Tg(E_3)$ is, abstractly, the group of rotations of a circle. Moreover, if K denotes such a group, the space E_3 must admit a coordinate system in which K is the group of all rigid rotations of E_3 about a fixed axis. This is now complemented by the following theorem: The only *non-abelian* $Tg(E_3)$ is, abstractly, the group of rotations of the two-sphere. Moreover, if G' denotes such a group, the space E_3 must admit a coordinate system in which G' is the group of all rigid rotations of E_3 about the origin. (Received June 16, 1938.)

304. A. R. Schweitzer: *An outline of the history and the philosophy of the concept of orientation.* Preliminary report.

Fundamental concepts of this paper are Reason and the Hand. The author analyzes critically various interpretations of orientation in ancient and modern literature and attempts to coordinate them in a unified whole with reference to their mathematical implications. The subject matter falls roughly into three categories: I, mystical speculation and theology; II_1, philosophy, metaphysics, psychology, and logic; II_2, aesthetics, art; III, science, mathematics. The corresponding main sources are: I, the scriptures and the works of Galen; II_1, Kant, early treatises and treatise on subjective orientation (1786), "Gestalt" psychologies of von Ehrenfels, Koehler, and Koffka; II_2, Leonardo da Vinci; III, Fresnel and Pasteur in science and Moebius, Grassmann, Klein, and Study in mathematics. Among the interpretations of orientation considered, are: right and left, mirror image, symmetry, balance; position, order; pointing, direction, polarity; motion, twist; inversion, asymmetry, and so on. The principal subjects considered are: in science, crystallography and cytology; physics, chemistry; biology, pathology; and in mathematics, geometry and topology. Orientation as such has been studied by G. Reynaud (1898), T. A. Cook (1914), F. M. Jaeger (1920), G. W. Crile (1926), W. Ludwig (1932), C. W. R. Hooker (1934), and others. In connection with subjective orientation, the author refers to his article in Revue de Metaphysique et de Morale, vol. 22 (1914). (Received June 11, 1938.)

305. A. H. Taub: *Spin representation of conformal groups.*

The conformal group of a flat n-space is isomorphic with the orthogonal group of a flat $(n+2)$-space. By using the spinor representation of the latter, a spin representation of the conformal group may be obtained. Also, a consideration of the form of the first isomorphism in detail readily yields, in the $(n+2)$-space, the images of rotations, translations, inversions, and dilations in the n-space. These make it possible to find the explicit forms of the spin transformations corresponding to each of the different types of transformations in the n-space. (Received June 18, 1938.)

306. A. H. Taub: *Tensor equations equivalent to the Dirac equations.*

When the identities satisfied by the matrices which occur in the Dirac equation are used, the tensor equations equivalent to the Dirac equation obtained by E. T. Whittaker (*On the relations of the tensor-calculus to the spinor-calculus*, Proceedings of the Royal Society, vol. 158 (1937), pp. 38–46) follow immediately. Another set of tensor equations is also obtained. These equations contain the current vector, and the divergence of an anti-symmetric tensor which describes the magnetic moment of the electron. It is also shown that the Dirac equations can be written in an equivalent spinor form. This form is used to calculate the "acceleration" in the Dirac theory, and it is shown to be equal to the divergence of a symmetric second order tensor which is similar to the stress energy tensor of the Maxwell theory. (Received June 18, 1938.)

307. W. L. Williams: *Permanent configurations in the problem of five bodies.*

A permanent configuration is a configuration of n bodies which has the property that the ratio of distances between corresponding bodies is constant. In other words, the figure may change in size, but not in shape. In the present paper, necessary and sufficient conditions for any plane configuration of five bodies (other than that in which all are in a straight line) are given, and also a detailed analysis of the various types of such configurations, both convex and concave, that can be formed by rearranging the bodies to give pentagons of different shapes. Several numerical examples are worked out, including the regular pentagon with one mass at each vertex and the square with one mass at each corner and the fifth mass at its center. (Received June 10, 1938.)

308. Reinhold Baer: *The significance of the system of subgroups for the structure of the group.*

If G and H are isomorphic groups, then their sets of subgroups are isomorphic and every isomorphism of G upon H induces an isomorphism of the set of subgroups of G upon the set of subgroups of H. If, conversely, G is an abelian group and G and H have isomorphic sets of subgroups, then G and H are isomorphic groups, provided the correspondence between the sets of subgroups preserves normality and indices. If G is a primary hamiltonian group of "not too small" an abelian group, then it is even possible to prove that every isomorphism of the set of subgroups of G is induced by an isomorphism of the group G. (Received June 27, 1938.)

309. E. F. Beckenbach and Maxwell Reade: *A characterization of minimal surfaces.*

Continuing the work reported in abstract 44-3-92, the authors prove the following theorem: If $x_j(u, v)$, $(j=1, 2, 3)$, are of class C_3 (that is, all derivatives of order less

than or equal to three are continuous) in a domain D, then a necessary and sufficient condition that these functions be the coordinate functions of a minimal surface given in (harmonic and) isothermic representation is that for each circle C lying in and enclosing only points of D, $\sum_{j=1}^{3}[\int_C x_j(u,v)(du+idv)]^2=0$. (Received June 24, 1938.)

310. Archie Blake: *Criteria for the reality of apparent periodicities and other regularities.* Preliminary report.

The following procedure is suggested to provide a more efficient statistic than the classical periodogram in the case of phenomena which are not nearly sinusoidal in type. Denote the observed function (after a suitable change of scale, if necessary) by $f(t)$, and let there be given also a weight function $w(t)$ and a period T to be tested. Compute the departure $d(t)$ by referring f to its mean. As a measure of the covariance, or contribution of the observations at any two times t_1 and t_2 to the periodicity, adopt a function $C(d(t_1), d(t_2); w(t_1), w(t_2); t_1, t_2; T)$, and define the periodicity $P(T)$ as the double integral of C with respect to t_1 and t_2. The function C is to be chosen to suit the needs of the type of problem in hand; there is considerable latitude here, by which such phenomena as quasi-persistence, seismic aftershocks, and line spectra can be appropriately treated. For the detection of certain other preassigned types of regularity than pure periodicity, the method of analysis is essentially the same, the parameters under investigation being used as arguments of C in place of T. (Received June 21, 1938.)

311. Jesse Douglas: *The most general form of the problem of Plateau.*

Suppose there is given any riemannian manifold R in the most general sense of the term, that is, any two-dimensional connected topological variety for which there is defined in the neighborhood of each point a local conformal representation on a circle. The manifold R may then have any finite or infinite number of boundaries, and any topological structure whatever, that is, any finite or infinite type of connectivity. It may also have either character of orientability, that is, one- or two-sidedness. Consider also any point set Γ in n-dimensional euclidean space which is a topological image of the total boundary C of R; the point set Γ may consist of any finite or infinite number of Jordan curves, together with their limit points; or it may be some more general type of point set. A definite sense of description is associated with each component of Γ, carried over from C, which may be supposed to be oriented so that R is on the left. The problem is to determine the existence of a minimal surface M topologically equivalent to R and bounded by Γ. This problem is completely solved in the present paper. A note will be published in the Proceedings of the National Academy of Sciences, and a more detailed presentation in the Journal of Mathematics and Physics. (Received June 28, 1938.)

312. Jesse Douglas: *Remarks on Riemann's doctoral dissertation.*

This paper will appear in full in the Proceedings of the National Academy of Sciences, July, 1938. (Received June 28, 1938.)

313. Aaron Fialkow: *Totally geodesic Einstein spaces.*

In this note, characteristic conditions under which an Einstein space E_m admits families of totally geodesic Einstein subspaces E_n are obtained. The results for the case $m>n+1$ have been stated by the author (cf. Proceedings of the National Academy of Sciences, vol. 24 (1938)) and are related to the problem of imbedding an E_m as a hypersurface of a space of constant curvature. If $m=n+1$, theorems such as

the following are obtained: A one-parameter family of isometric E_n's can be imbedded as ∞^1 non-parallel totally geodesic hypersurfaces of an E_{n+1} if and only if each E_n can be mapped conformally on another Einstein space by means of a function σ with $\Delta_1\sigma \neq 0$. If a and b are the mean curvatures of E_{n+1} and E_n, respectively, then $nb=(n-1)a$. (Received June 28, 1938.)

314. O. H. Hamilton: *Some theorems concerning points and continua left invariant by transformations of continua not locally connected.*

The following results are obtained. If M is a compact continuum in the plane S which does not separate S, then every reversibly continuous transformation of M into a subset of itself leaves invariant some point, some continuum which is the sum of a simply connected domain and its boundary, or some continuum whose boundary is indecomposable. If M, in addition to satisfying the requirements listed above, does not contain an indecomposable continuum in its boundary, then every reversibly continuous transformation of M into itself leaves some point invariant. Finally, these theorems, with modifications, are generalized for continua in a metric space. (Received June 23, 1938.)

315. Archibald Henderson and J. W. Lasley: *On harmonic separation.*

The authors present an historical account of the development of harmonic separation from the time of the ancient Greeks to the present. The steps in the evolution of the linear construction for harmonic conjugates are outlined from Serenus to Chasles. Other harmonic constructions are given; some involving two circles, some one circle. The notions of inversion and of similar triangles are employed. Analytic methods applied to the bisector problem and to the circle of Apollonius yield expressions for the foregoing geometric solutions in algebraic form. (Received June 27, 1938.)

316. E. V. Huntington: *Note on a recent set of postulates for the calculus of propositions.*

In my paper, *Postulates for assertion, conjunction, negation, and equality* (Proceedings of the American Academy of Arts and Sciences, vol. 72 (1937), pp. 1–44), in which the postulates 1–17 were expressed, after Lewis, in terms of the base K, T, $\times$, $'$, $\equiv$, the definition $a \dashv b :=: ab' \equiv Z$ may be replaced by the simpler form $a \dashv b :=: ab \equiv a$ provided the theorem $a \dashv b . =. b' \dashv a'$ is added to the list of postulates. In the revised list thus obtained, the three special elements Z, Z', Z^* appear only by inference, no explicit mention of them in the postulates being required. (Received June 25, 1938.)

317. N. H. McCoy: *Generalized regular rings.*

Let $\mathfrak{R}$ be a ring with unit element. If for every element a of $\mathfrak{R}$ there exists a positive integer n such that a^n is regular (von Neumann, Proceedings of the National Academy of Sciences, vol. 22 (1936), pp. 707–713, it is said that $\mathfrak{R}$ is *π-regular*. If there is a fixed integer m such that for all a in $\mathfrak{R}$, a^m is regular, $\mathfrak{R}$ is said to be *m-regular*. The purpose of the present note is to prove a few theorems concerning π-regular and m-regular rings. For the most part, they are analogs of known theorems on regular rings. (Received June 23, 1938.)

318. E. W. Paxson: *Induced norms in topological groups.*

It is shown that, in the presence of connectedness, an extension of the concept of convexity, and sequential completeness, the notions of linearity and norm as ideas

relative to a designated neighborhood of the identity may be induced in a space that supports simply a Hausdorff group structure. In a sense, these results supplement those of J. von Neumann (Transactions of this Society, vol. 37 (1935), pp. 1–20) on pseudo-metrics in linear topological spaces. (Received June 27, 1938.)

319. H. S. Carslaw and J. C. Jaeger: *On Green's functions in the theory of heat conduction.*

In this note, the second of the problems discussed by Lowan in his paper *On the operational determination of two dimensional Green's functions in the theory of heat conduction* (this Bulletin, vol. 44 (1938), pp. 125–133) is solved, first, by contour integration and, second, by the use of the Laplace transformation. The connection between the two methods is pointed out and an error in Lowan's result is corrected. (Received June 25, 1938.)

320. W. H. Gage: *Class number relations for the form* x^2-2y^2.

By means of the expansions for $\phi(x, y, z)$ (American Journal of Mathematics, vol. 59 (1937), pp. 919–920), and by the method of paraphrase, a number of the class number relations of Petr and Humbert have been obtained. These relations involve sums of ideal divisors of an integer $n=x^2-2y^2$. A number of new relations for $n-u^2=x^2-2y^2$, $n-u=x^2-2y^2$, n fixed and u variable, are also given. All relations are obtained by equating coefficients of q^n in identities involving theta expansions. Class numbers are introduced in the identities through the formulas for the numbers of representations of integers in the forms $xy+yz+zx$, $xy+yz+2zx$, $xy+2yz+2zx$. (Received July 1, 1938.)

321. Henry Gerhardt: *The generalization of Miquel's theorem.*

The purpose of this paper is to determine whether Miquel's theorem is valid in hyperbolic geometry. A conformal representation of the hyperbolic plane on the euclidean plane is used, with straight lines and circles in the former represented by half-circles and circles in the latter. It is found that Miquel's theorem is not valid in hyperbolic geometry, but a generalized form of the theorem is obtained which is valid in both hyperbolic and euclidean geometry. This method of generalizing theorems of euclidean geometry by conformal representation of the hyperbolic plane seems to be a fertile one. (Received June 25, 1938.)

322. J. L. Walsh and W. E. Sewell: *Note on degree of trigonometric and polynomial approximation to an analytic function.*

In this note the authors assume $f(z)$ to be analytic in the annulus $\rho>|z|>1/\rho<1$ and continuous in the corresponding closed region, and show that certain continuity properties (Lipschitz conditions on function or derivatives) of $f(z)$ on $|z|=\rho$ and $|z|=1/\rho$ imply a specific degree of convergence of various trigonometric polynomials and polynomials in z and $1/z$ to $f(z)$ on $|z|=1$. Conversely, it is shown that a given degree of convergence on $|z|=1$ implies certain continuity properties of $f(z)$ on $|z|=\rho$ and $|z|=1/\rho$. By well known transformations, these results are extended to a function $f(z)$ analytic in an ellipse whose foci are $+1$ and -1 and the semi-sum of whose axes is ρ, and to a function $f(z)$ periodic with period 2π and analytic in the band $|y|<\log\rho$, where $z=x+iy$. (Received July 1, 1938.)

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